

Amplituhedra and Origami

Pavel Galashin (Cornell/UCLA)

University of Geneva Mathematics Colloquium

March 11, 2026

[arXiv:2410.09574](#), [arXiv:26XX.XXXXX](#)

≡ Kawasaki's theorem

🌐 6 languages ▾

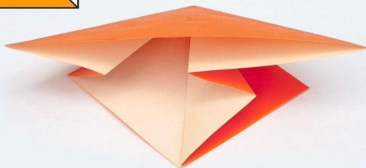
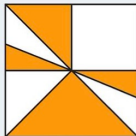
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Kawasaki's theorem or **Kawasaki–Justin theorem** is a [theorem](#) in the [mathematics of paper folding](#) that describes the [crease patterns](#) with a single [vertex](#) that may be folded to form a flat figure. It states that the pattern is flat-foldable if and only if alternately adding and subtracting the angles of consecutive folds around the vertex gives an [alternating sum](#) of zero. Crease patterns with more than one vertex do not obey such a simple criterion, and are [NP-hard](#) to fold.



In this example, the alternating sum of angles (clockwise from the bottom) is $90^\circ - 45^\circ + 22.5^\circ - 22.5^\circ + 45^\circ - 90^\circ + 22.5^\circ - 22.5^\circ = 0^\circ$. Since it adds to zero, the crease pattern may be flat-folded. 

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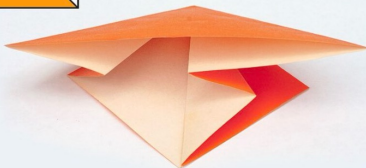
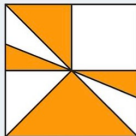
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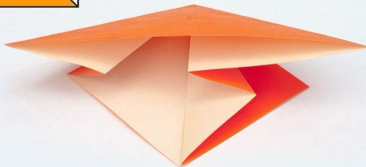
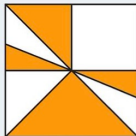
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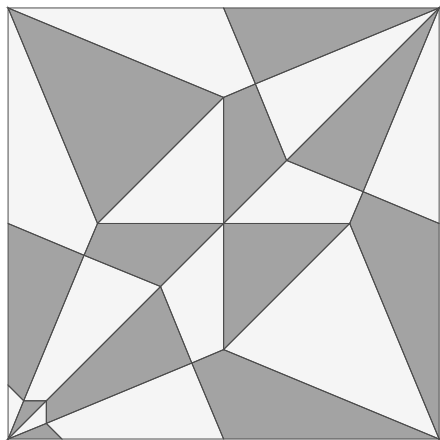


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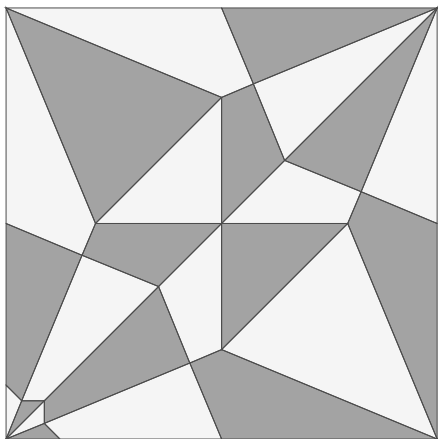


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Origami crease patterns (OCP)

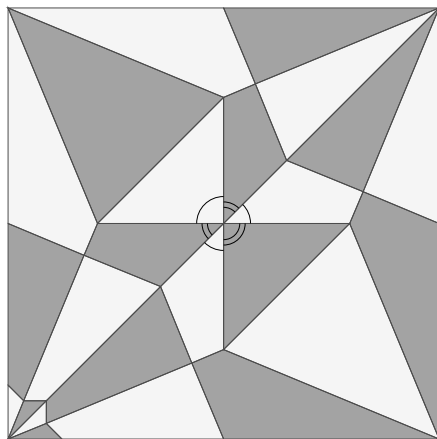


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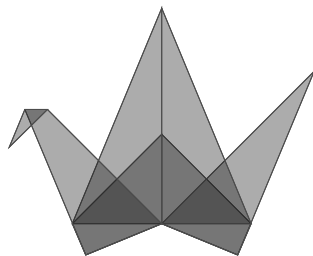
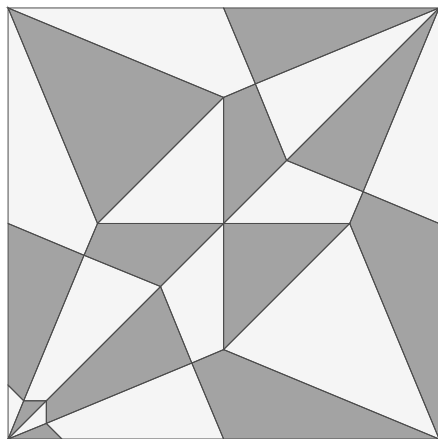
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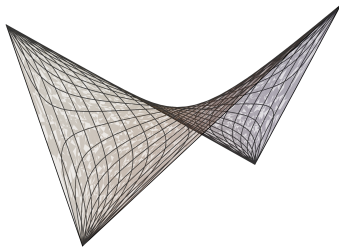
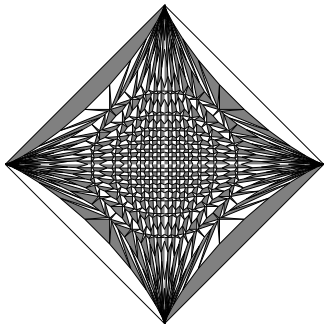
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- Faces: convex polygons colored black and white;
- Kawasaki angle condition: $\text{sum}(\text{white angles}) = \text{sum}(\text{black angles}) = \pi$ around each interior vertex.
- **Origami map**: isometry on each face preserving/reversing the orientations of white/black faces.

Convergence results for dimer model observables on OCPs:
[Kenyon–Lam–Ramassamy–Russkikh '18], [Chelkak–Laslier–Russkikh '21]



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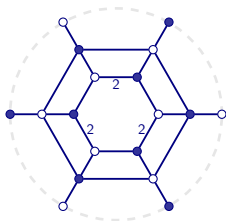
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Conjecture (OCP Existence Conjecture [KLRR '18], [CLR '21])

Every weighted planar bipartite graph Γ can be realized geometrically* as a planar dual of an embedded OCP.



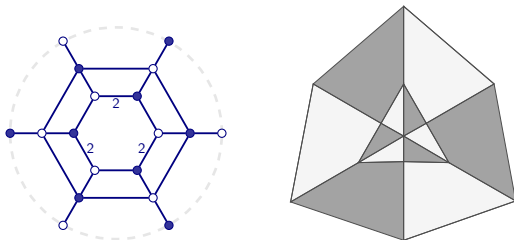
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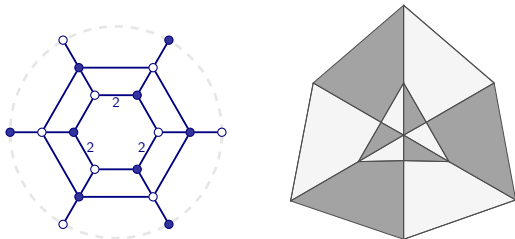
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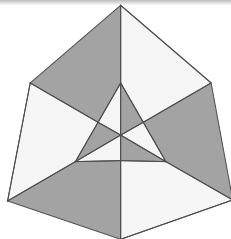
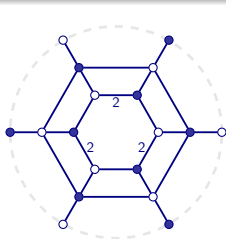
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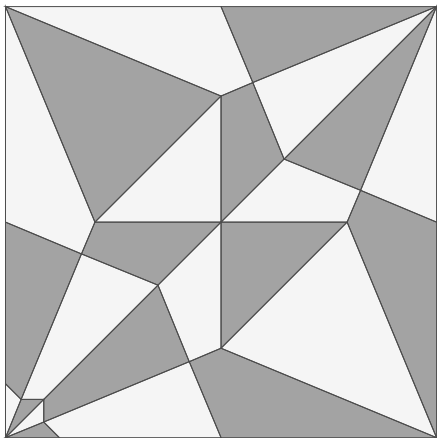
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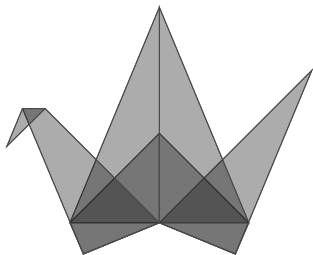
Proof.

Total positivity + physics of scattering amplitudes. □



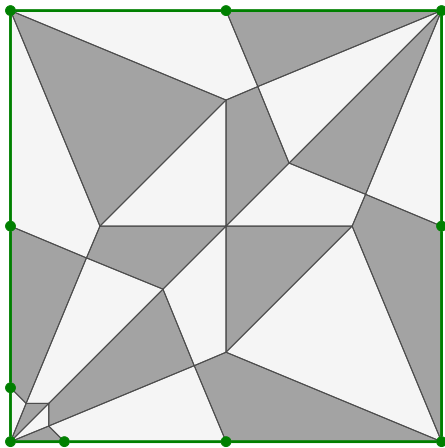


Kami plane ($\mathbb{R}^2$)

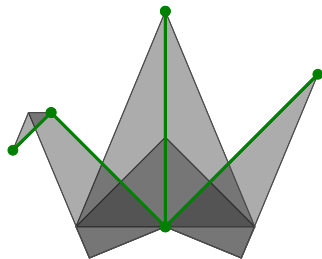


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- Denote boundary vertices by $\hat{x}_1, \dots, \hat{x}_n \in \mathbb{R}^2$ and $\check{x}_1, \dots, \check{x}_n \in \mathbb{R}^2$.

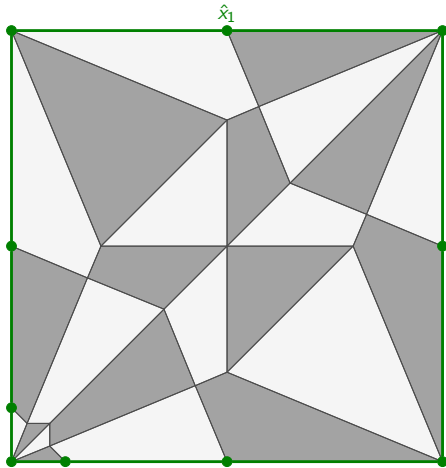


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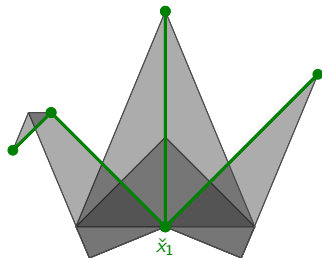


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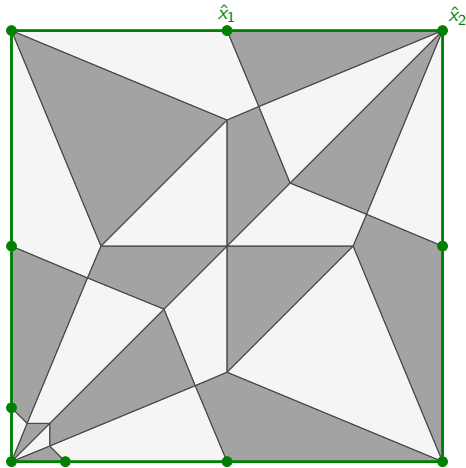


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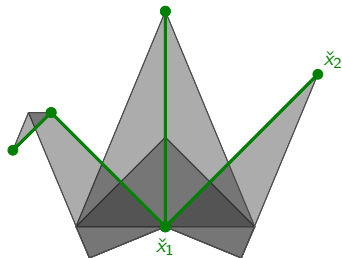


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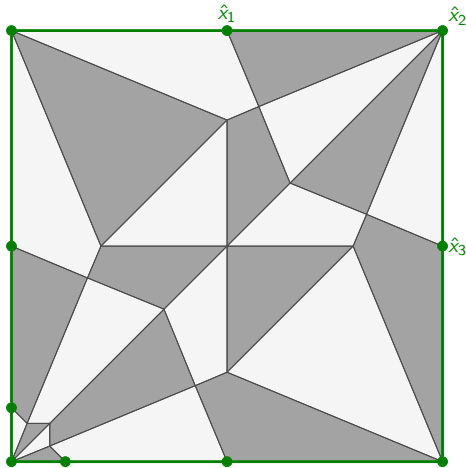


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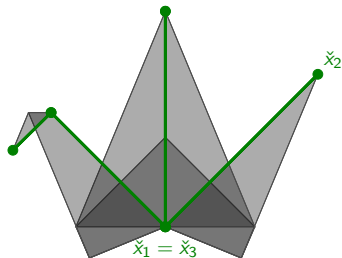


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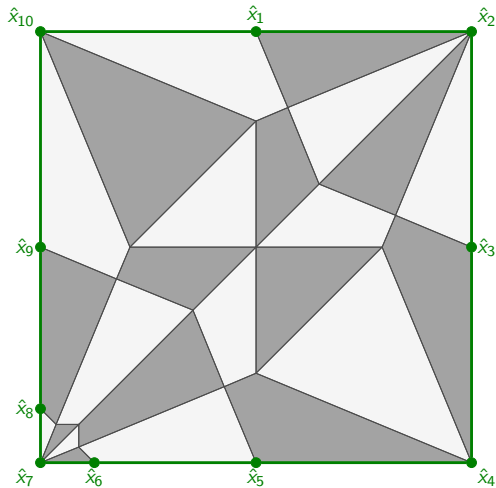


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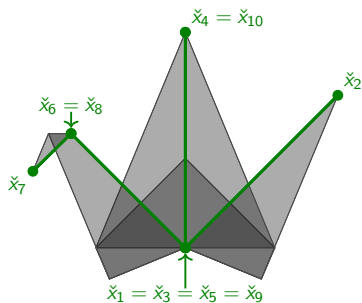


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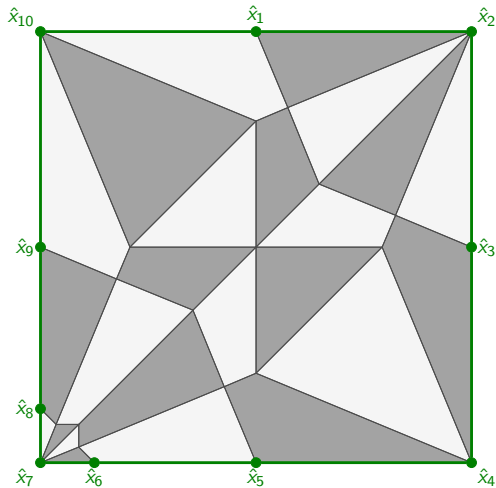


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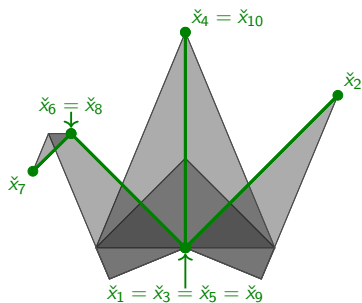


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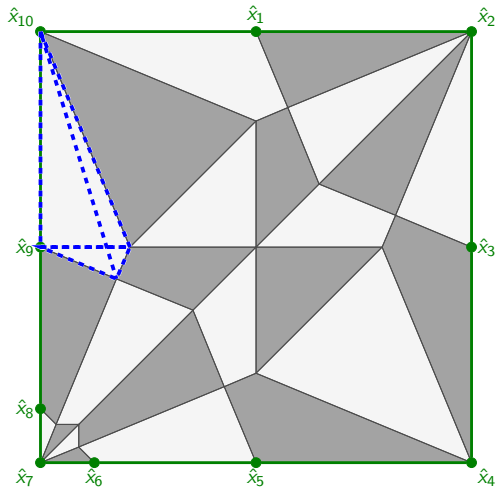


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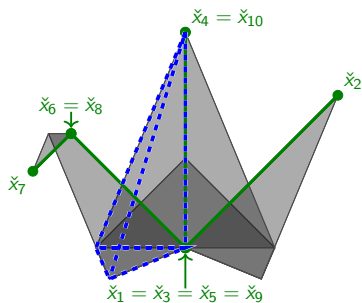


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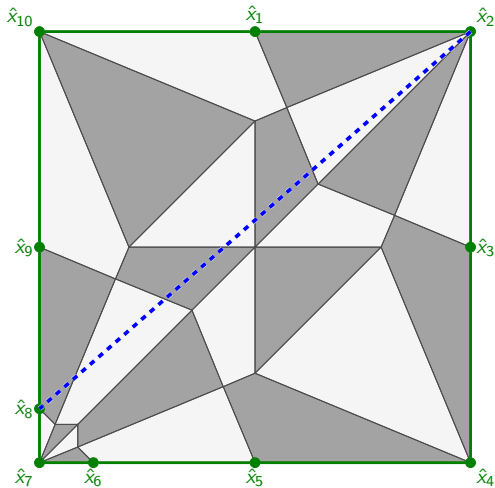


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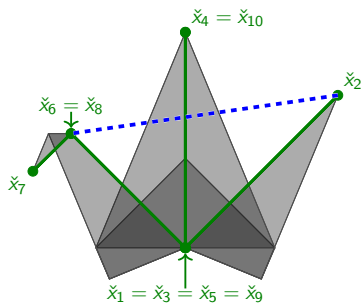


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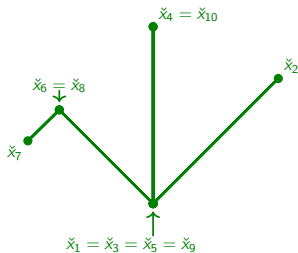
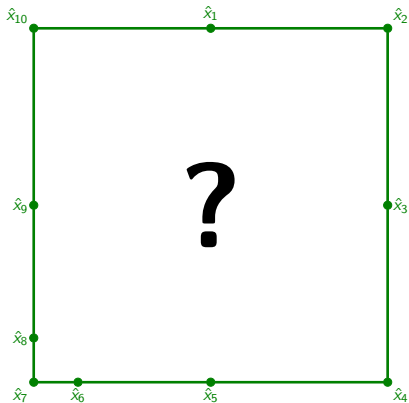


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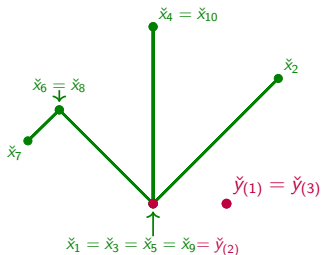
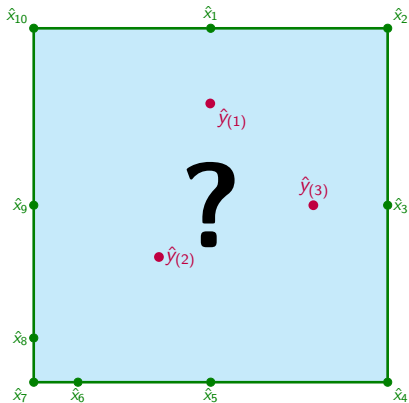
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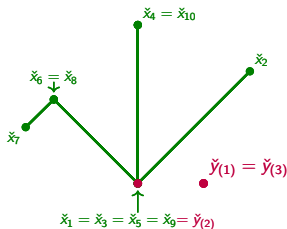
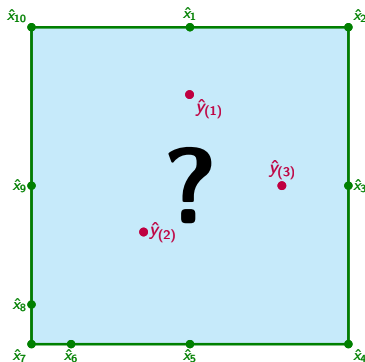
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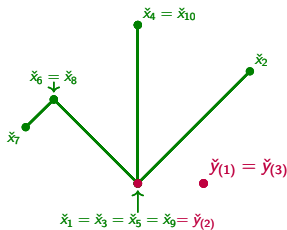
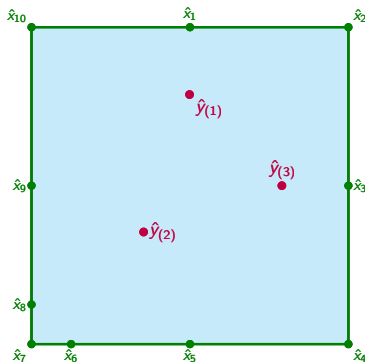
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- **L-punctured origami reconstruction problem:** same as above but we additionally require the origami map to send $(\hat{y}_{(1)}, \dots, \hat{y}_{(L)}) \mapsto (\check{y}_{(1)}, \dots, \check{y}_{(L)})$.



L-punctured origami reconstruction problem: Find an OCP with boundary $(\hat{x}_1, \dots, \hat{x}_n)$ such that the origami map sends $\hat{x}_i \mapsto \check{x}_i$ and $\hat{y}_{(j)} \mapsto \check{y}_{(j)}$ for all i, ρ .

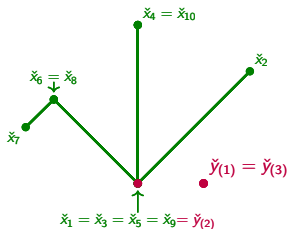
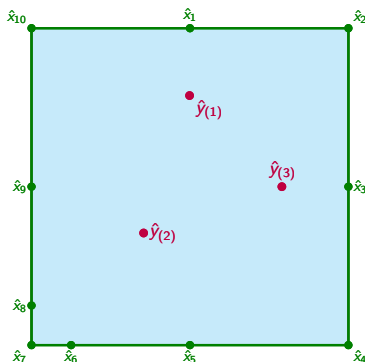


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Origami Reconstruction Algorithm

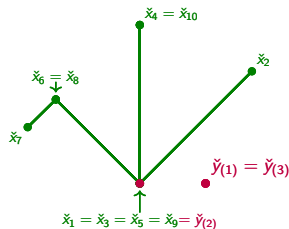
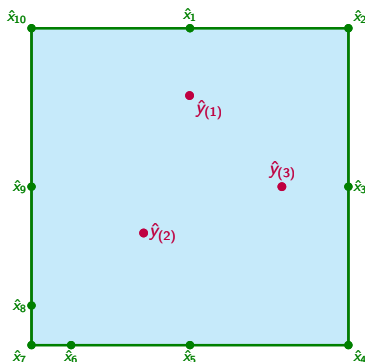
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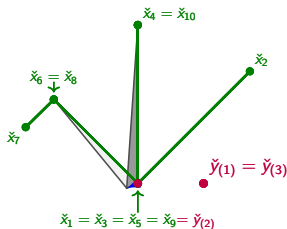
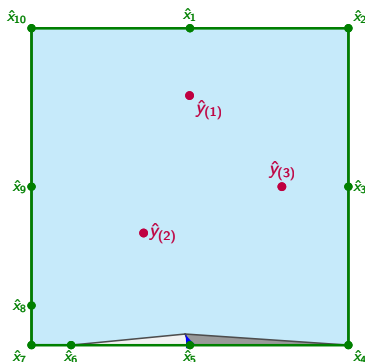
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Origami Reconstruction Algorithm

- 1 Choose $1 \leq i \leq n$ and a convention ($\circ \bullet$ vs $\bullet \circ$).
- 2 Find the unique point $\hat{x}_i(t)$ that forms white/black triangles with $\hat{x}_{i-1}, \hat{x}_i, \hat{x}_{i+1}$ and satisfies $|\hat{x}_i(t) - \hat{u}| = |\check{x}_i(t) - \check{u}|$ for exactly one other vertex $\hat{u}$.
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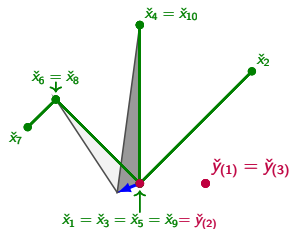
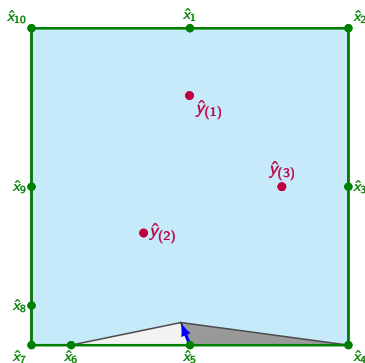
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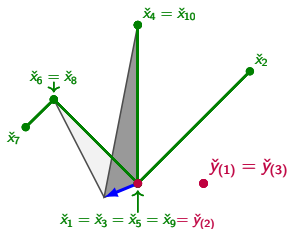
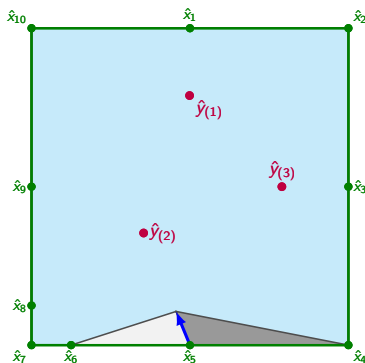
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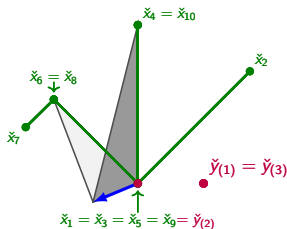
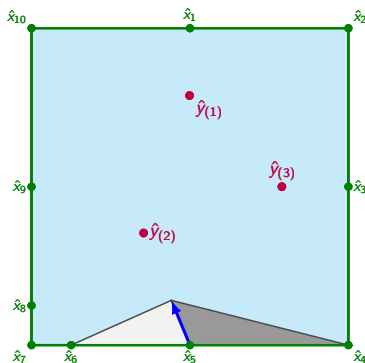
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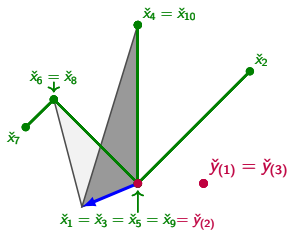
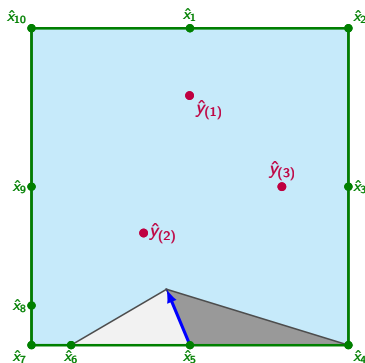
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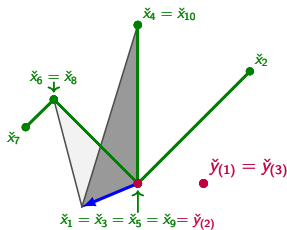
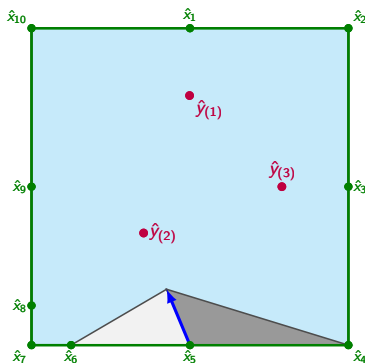
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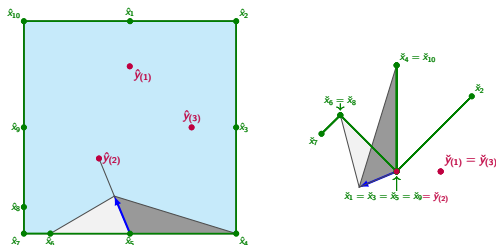
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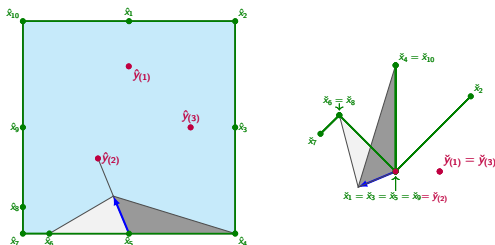
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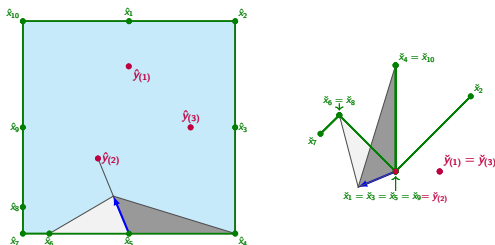
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Theorem (G. '26+)

The Origami Reconstruction Algorithm *works*.

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Origami Reconstruction Algorithm

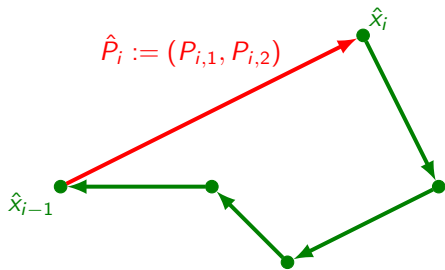
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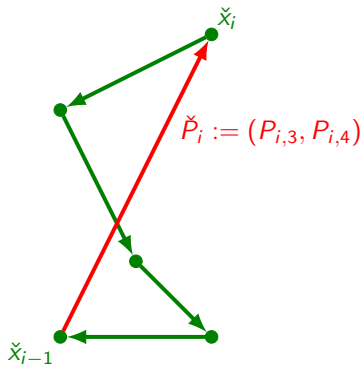
The Origami Reconstruction Algorithm works.

For all inputs $(x_1, \dots, x_n), (y_{(1)}, \dots, y_{(L)})$, it outputs a **valid, embedded OCP**.

- **Input:** $P_1, P_2, \dots, P_n \in \mathbb{R}^{2,2}$, **massless:** $P_i^2 := P_{i,1}^2 + P_{i,2}^2 - P_{i,3}^2 - P_{i,4}^2 = 0$,
momentum conservation: $P_1 + P_2 + \dots + P_n = 0$.



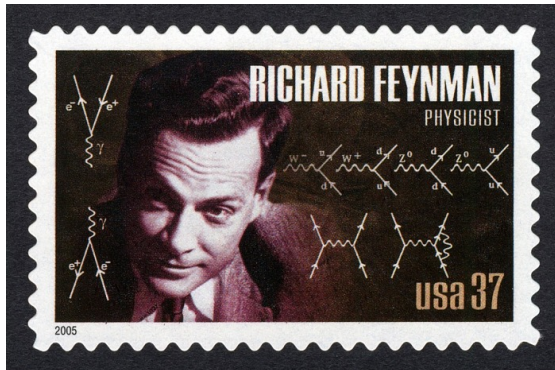
Space ($\mathbb{R}^2$)



Time ($\mathbb{R}^2$)

- **Input:** $P_1, P_2, \dots, P_n \in \mathbb{R}^{2,2}$, massless: $P_i^2 := P_{i,1}^2 + P_{i,2}^2 - P_{i,3}^2 - P_{i,4}^2 = 0$, momentum conservation: $P_1 + P_2 + \dots + P_n = 0$.
- **Output:** Scattering amplitude

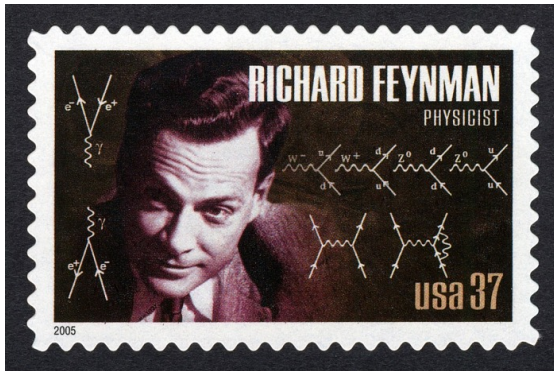
$$A(P_1, P_2, \dots, P_n) = \sum_{\text{Feynman diagram } D} A_D = A^{(L=0)} + A^{(L=1)} + A^{(L=2)} + \dots$$



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$L = |E| - |V| + 1$



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[Britto–Cachazo–Feng–Witten '05]

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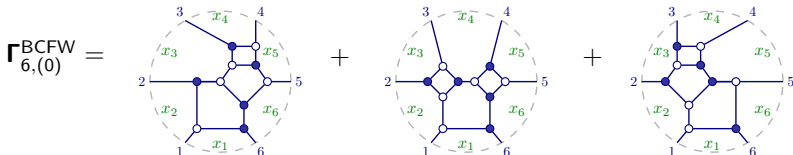
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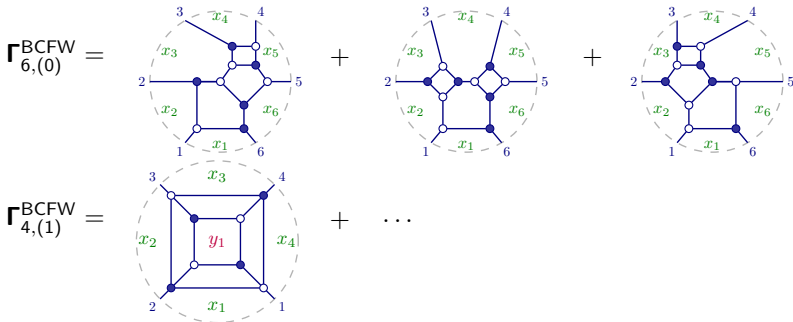
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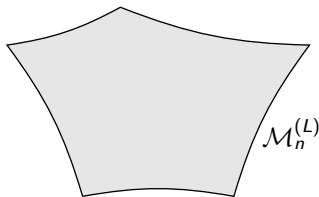
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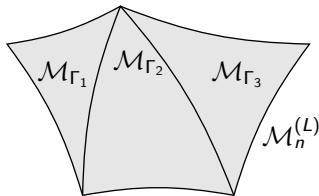
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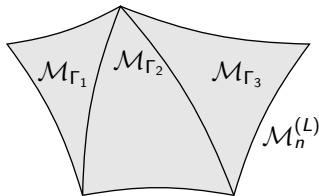
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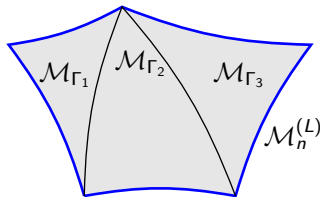
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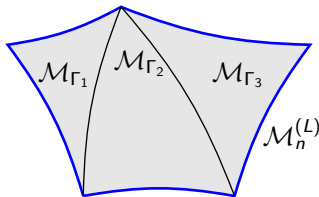
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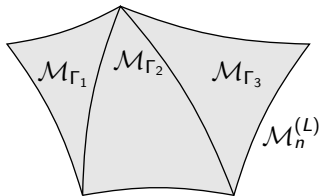
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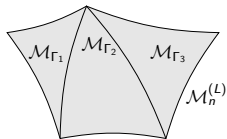


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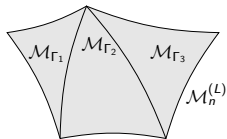
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Conjecture (BCFW Triangulation Conjecture [AHT'13])

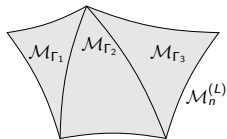
The BCFW tiles triangulate $\mathcal{M}_n^{(L)}$.

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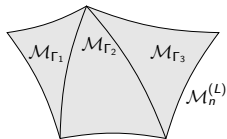
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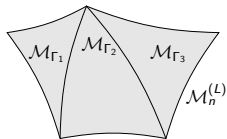
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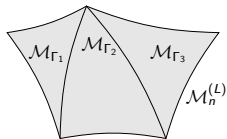
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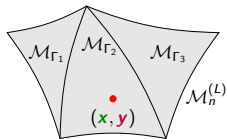
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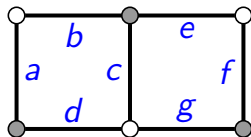
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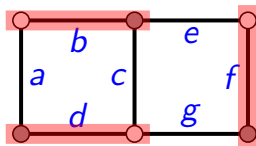
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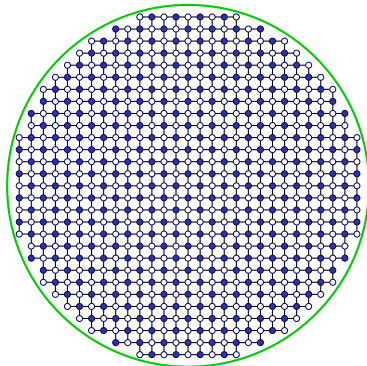
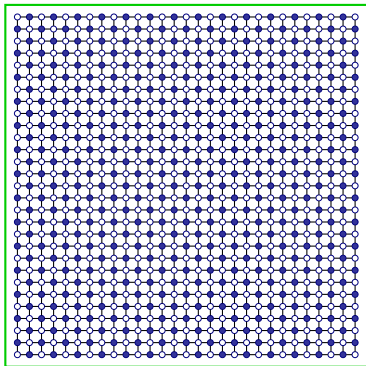
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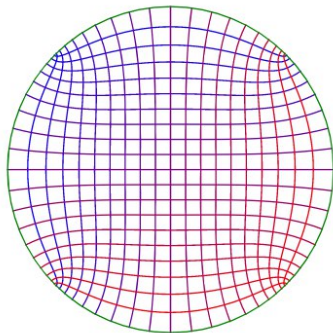
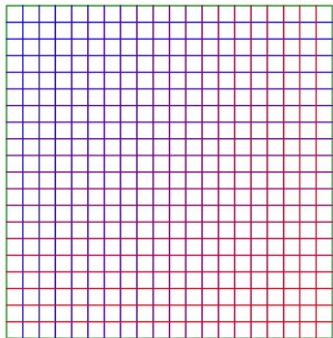
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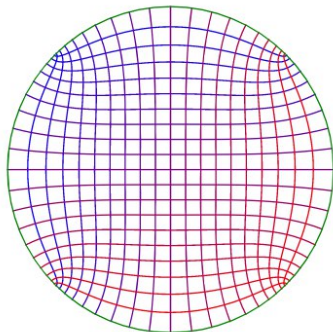
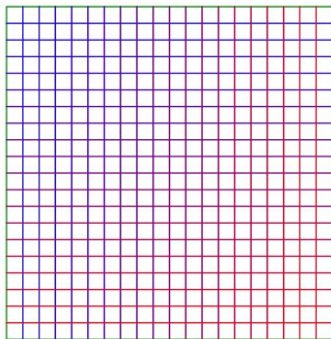
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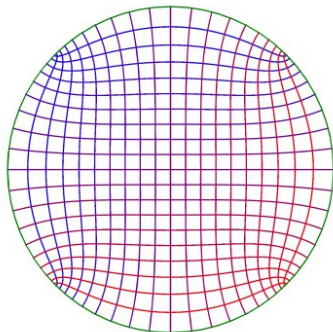
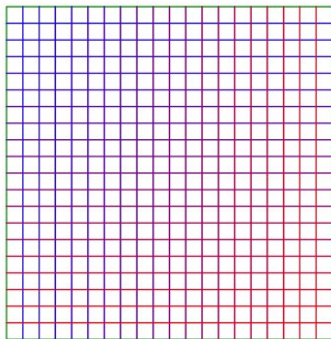
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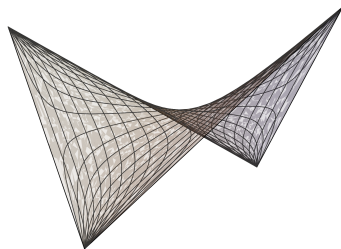
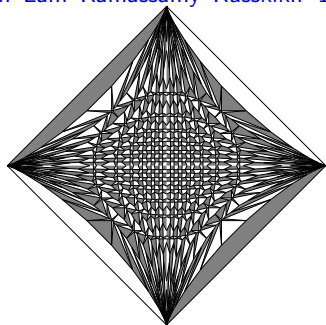
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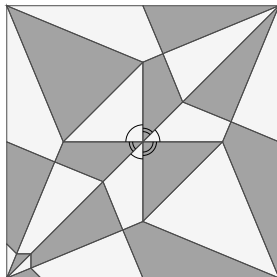


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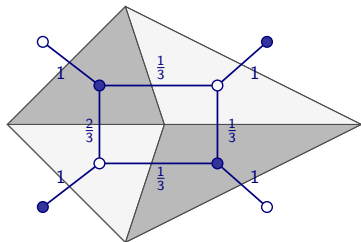
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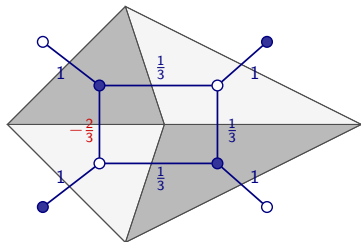
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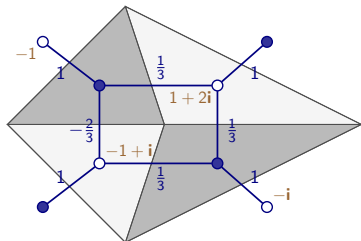
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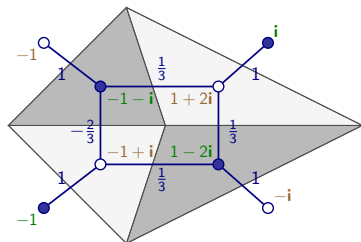
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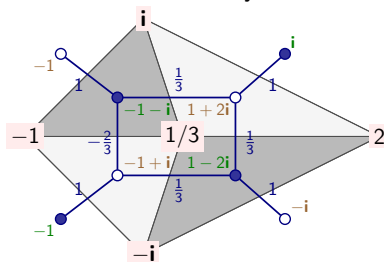
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Theorem (Origami–Amplituhedron Correspondence (G. 2024))

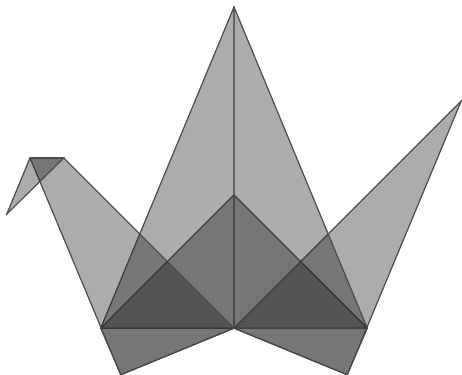
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*A Kenyon–Smirnov primitive gives rise to an embedded OCP $\iff$ its $\mathbb{R}^{2,2}$ -boundary polygon belongs to **momentum amplituhedron** $\mathcal{M}_n^{(L=0)}$.*



Thanks!