

Amplituhedra and Origami

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Richard P. Stanley Seminar in Combinatorics

September 23, 2026

[arXiv:2410.09574v2](#), [arXiv:2606.03439](#)



galashin.github.io/origami

≡ Kawasaki's theorem

🌐 6 languages ▾

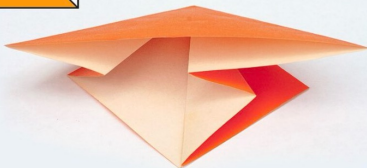
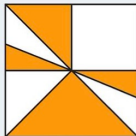
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Kawasaki's theorem or **Kawasaki–Justin theorem** is a [theorem](#) in the [mathematics of paper folding](#) that describes the [crease patterns](#) with a single [vertex](#) that may be folded to form a flat figure. It states that the pattern is flat-foldable if and only if alternately adding and subtracting the angles of consecutive folds around the vertex gives an [alternating sum](#) of zero. Crease patterns with more than one vertex do not obey such a simple criterion, and are [NP-hard](#) to fold.



In this example, the alternating sum of angles (clockwise from the bottom) is $90^\circ - 45^\circ + 22.5^\circ - 22.5^\circ + 45^\circ - 90^\circ + 22.5^\circ - 22.5^\circ = 0^\circ$. Since it adds to zero, the crease pattern may be flat-folded. 

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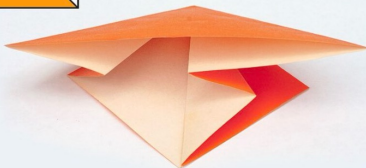
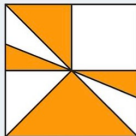
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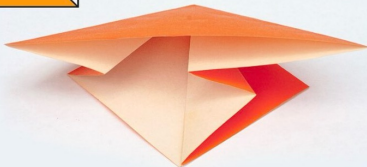
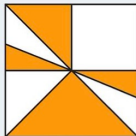
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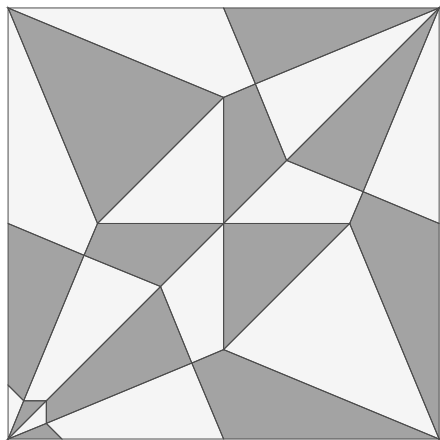


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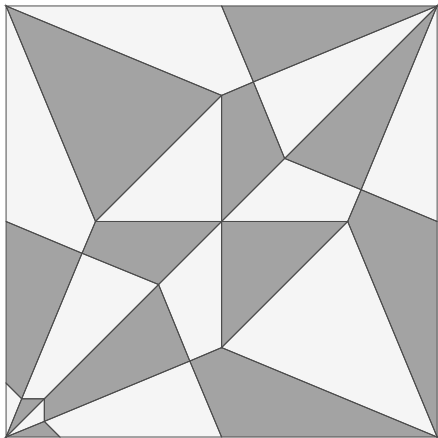


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Origami crease patterns (OCP)

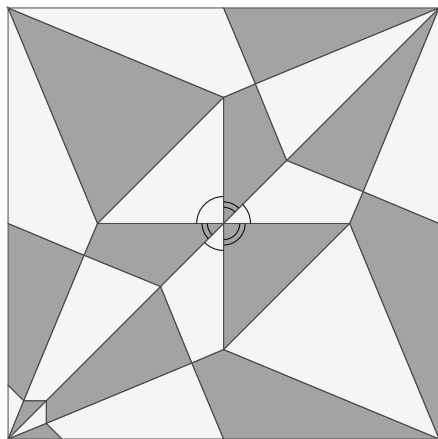


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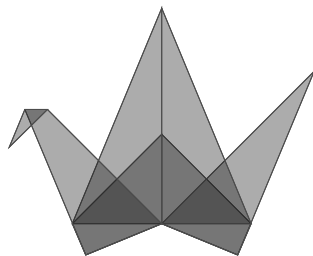
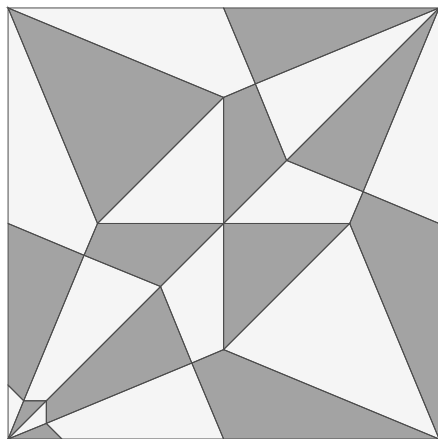
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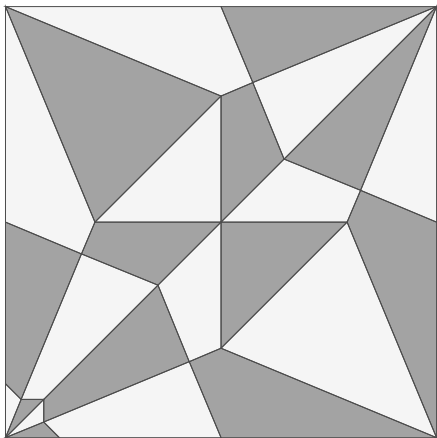


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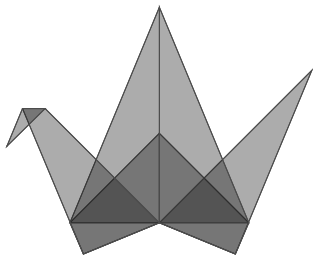
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- **Origami map**: isometry on each face preserving/reversing the orientations of white/black faces.

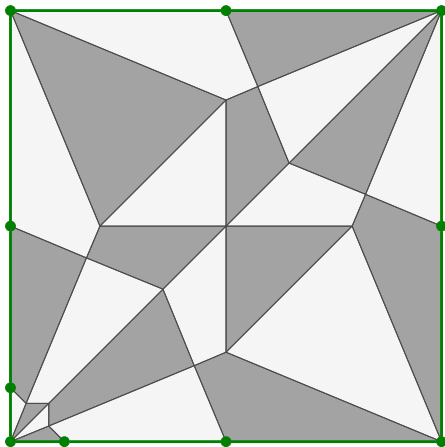


“Space” ($\mathbb{R}^2$)

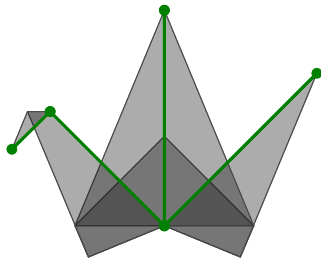


“Time” ($\mathbb{R}^2$)

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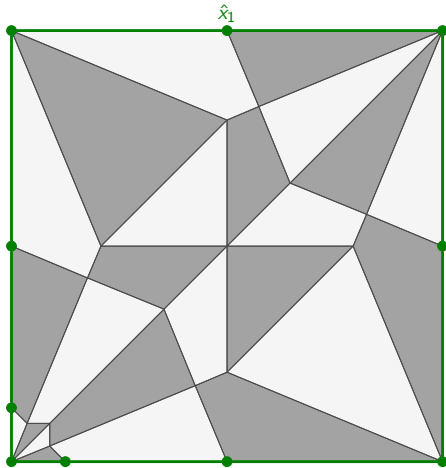


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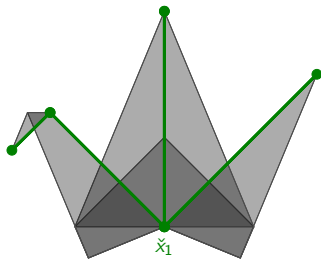


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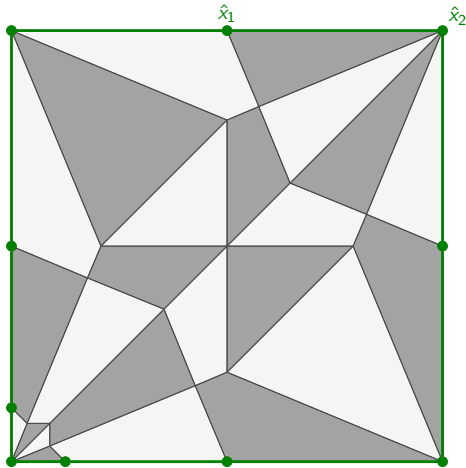


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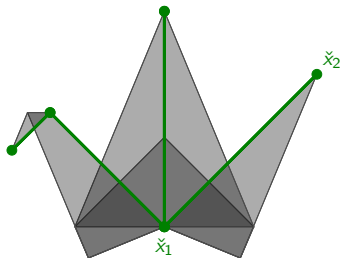


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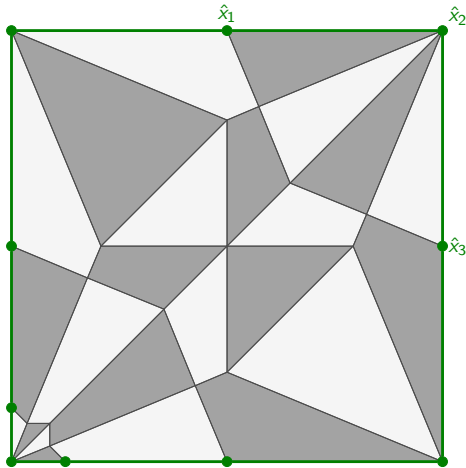


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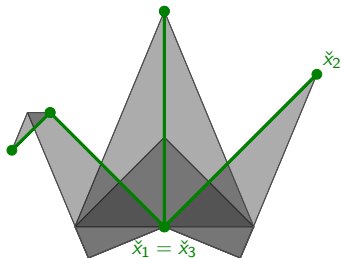


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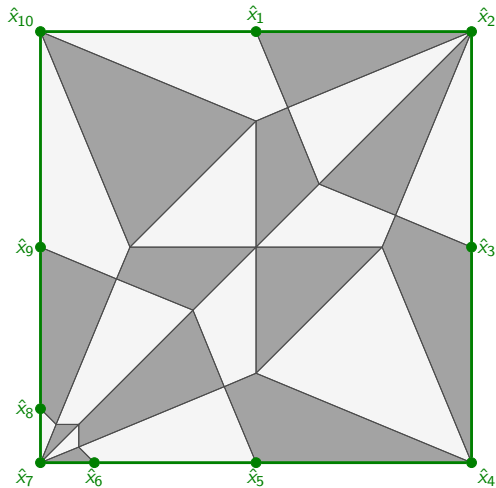


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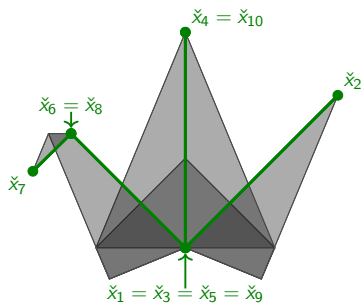


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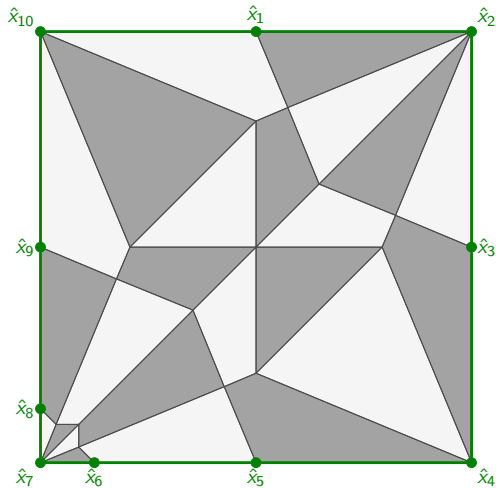


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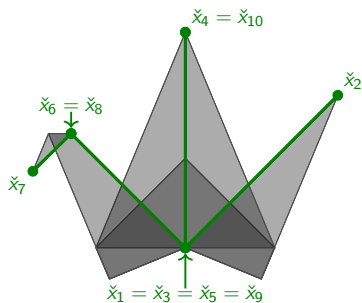


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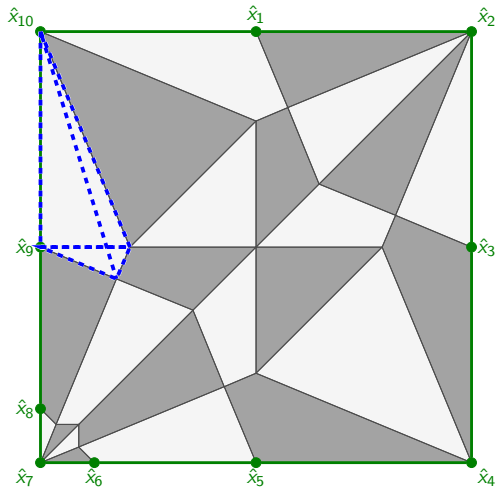


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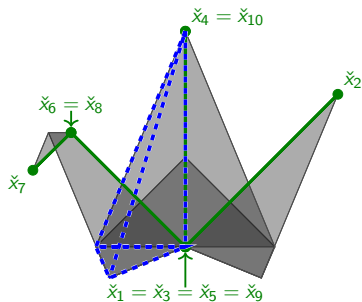


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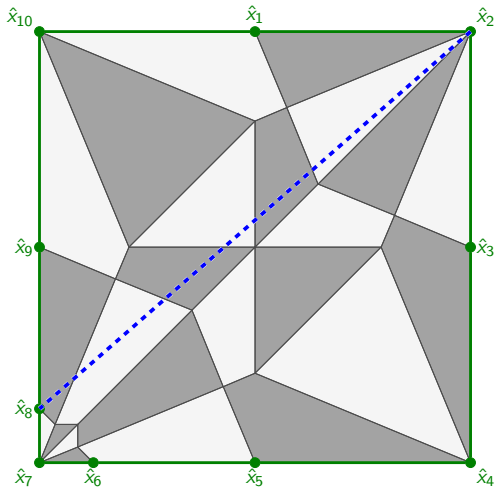


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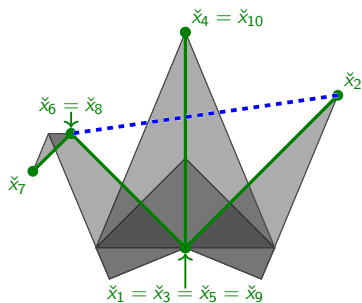


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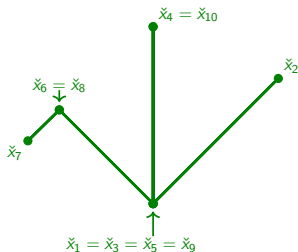
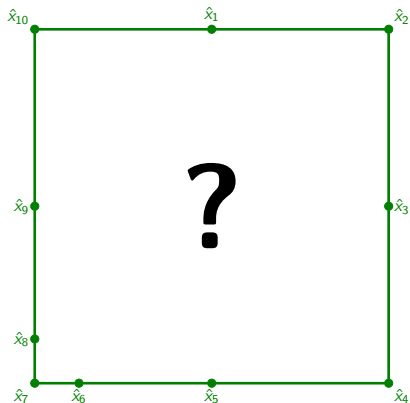


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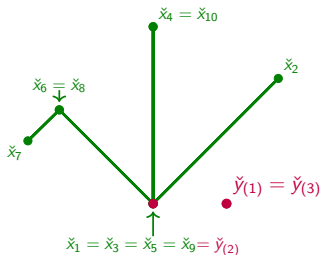
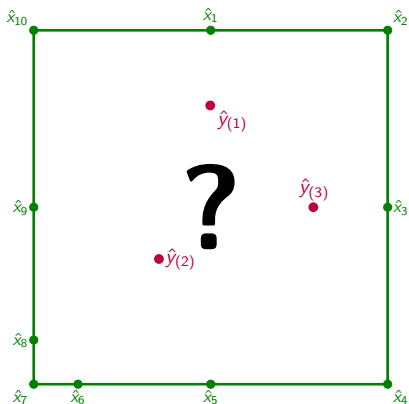
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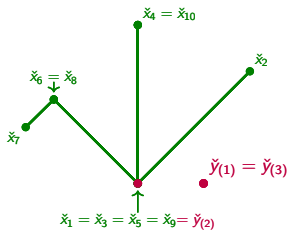
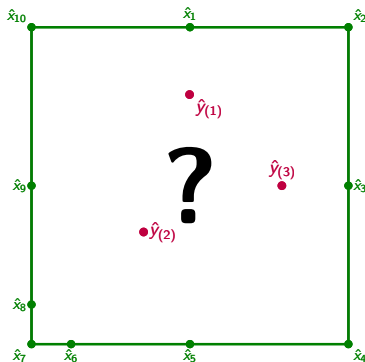
- M-positivity: origami map does not increase distances: $|\hat{x}_i - \hat{x}_j| \geq |\check{x}_i - \check{x}_j|$.
- **Origami reconstruction problem:** given points $(\hat{x}_1, \dots, \hat{x}_n)$ and $(\check{x}_1, \dots, \check{x}_n)$ satisfying $|\hat{x}_i - \hat{x}_j| \geq |\check{x}_i - \check{x}_j|$, find an OCP with boundary polygon $(\hat{x}_1, \dots, \hat{x}_n)$ whose origami map sends $\hat{x}_i \mapsto \check{x}_i$ for all i .



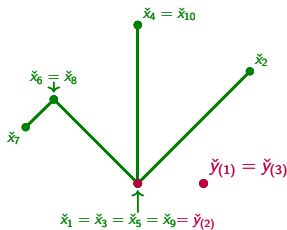
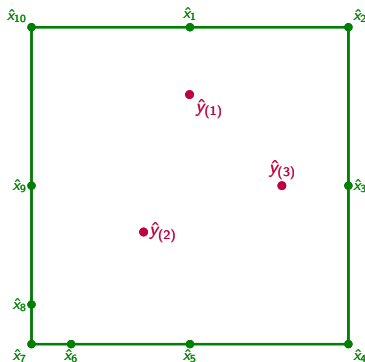
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- **L-punctured origami reconstruction problem:** same as above but we additionally require the origami map to send $(\hat{y}_{(1)}, \dots, \hat{y}_{(L)}) \mapsto (\check{y}_{(1)}, \dots, \check{y}_{(L)})$.



L-punctured origami reconstruction problem: Find an OCP with boundary $(\hat{x}_1, \dots, \hat{x}_n)$ such that the origami map sends $\hat{x}_i \mapsto \check{x}_i$ and $\hat{y}_{(j)} \mapsto \check{y}_{(j)}$ for all i, ρ .

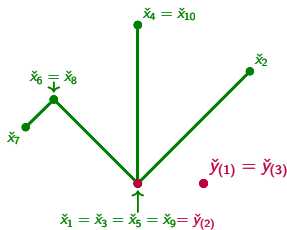
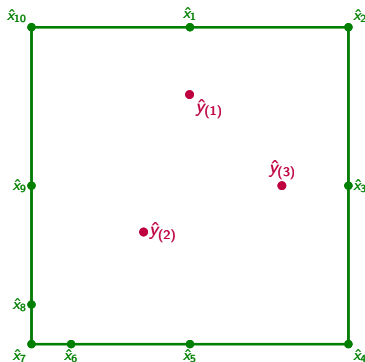


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Origami Reconstruction Algorithm

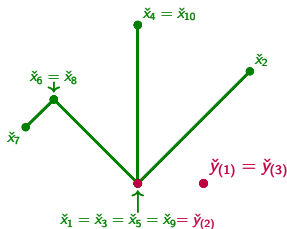
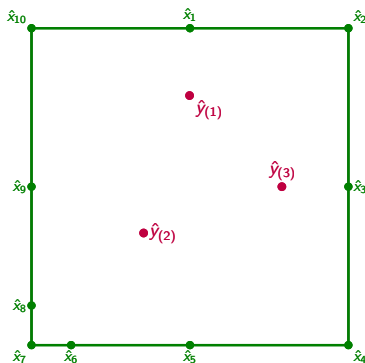
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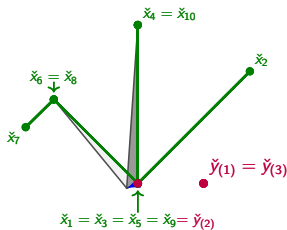
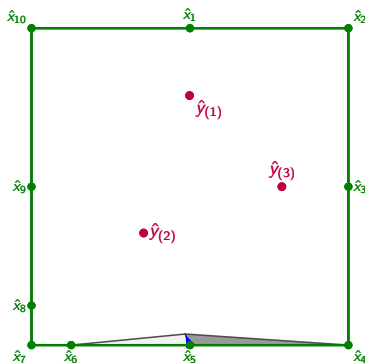
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Origami Reconstruction Algorithm

- 1 Choose $1 \leq i \leq n$ and a convention ($\circ \bullet$ vs $\bullet \circ$).
- 2 Find the unique point $\hat{x}_i(t)$ that forms white/black triangles with $\hat{x}_{i-1}, \hat{x}_i, \hat{x}_{i+1}$ and satisfies $|\hat{x}_i(t) - \hat{u}| = |\check{x}_i(t) - \check{u}|$ for exactly one other vertex $\hat{u}$.
(and $|\hat{x}_i(t) - \hat{v}| \geq |\check{x}_i(t) - \check{v}|$ for all other vertices $\hat{v}$)

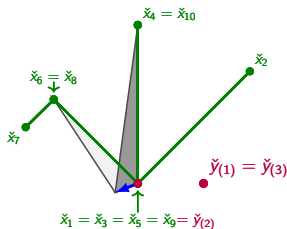
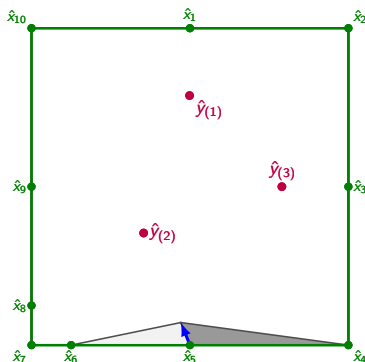
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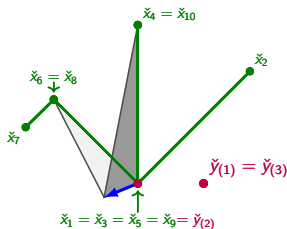
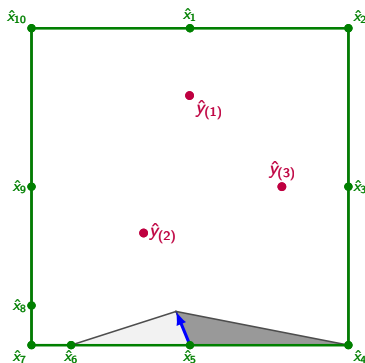
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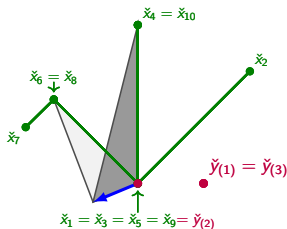
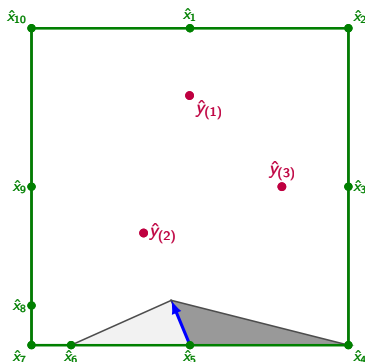
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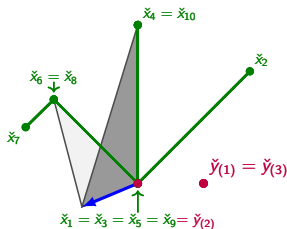
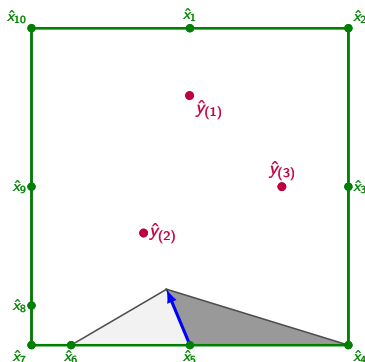
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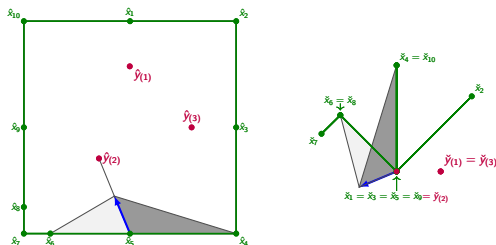
L-punctured origami reconstruction problem: Find an OCP with boundary $(\hat{x}_1, \dots, \hat{x}_n)$ such that the origami map sends $\hat{x}_i \mapsto \check{x}_i$ and $\hat{y}_{(j)} \mapsto \check{y}_{(j)}$ for all i, ρ .



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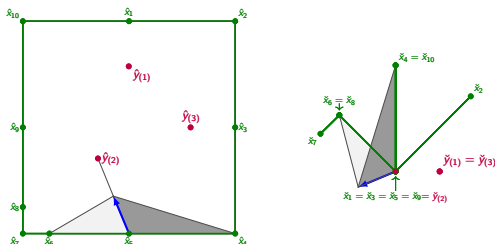
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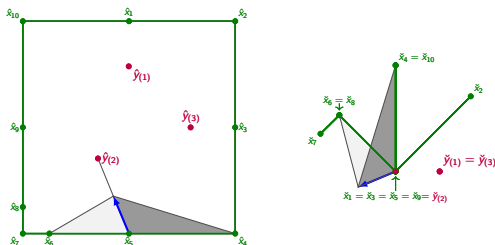
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The Origami Reconstruction Algorithm *works*.

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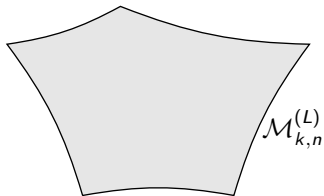
For all M -positive $(x_1, \dots, x_n)$, $(y_{(1)}, \dots, y_{(L)})$, it outputs a **valid, embedded** OCP.

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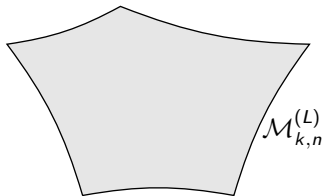
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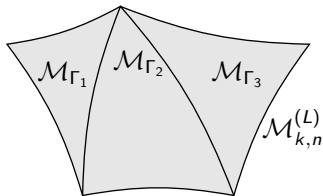
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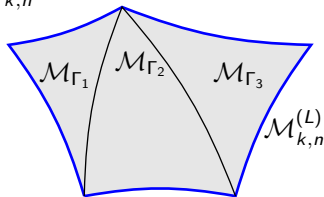
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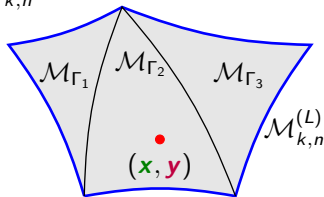


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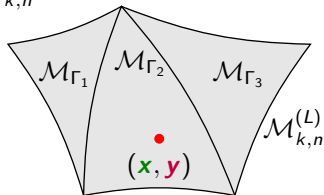
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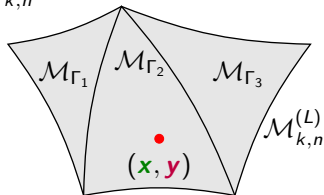
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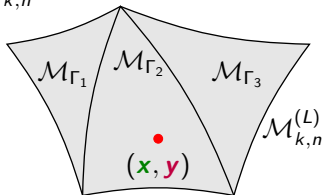
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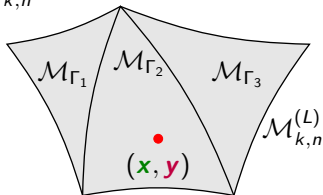
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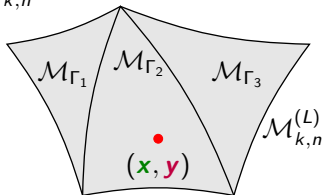
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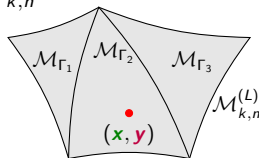
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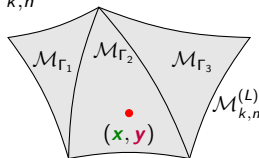
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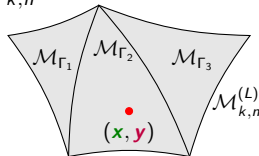
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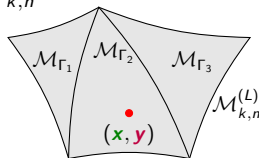
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Dimer model and positive Grassmannian

- $\text{Gr}(k, n) := \{C \subset \mathbb{R}^n \mid \dim C = k\}$

$$C = \begin{pmatrix} 1 & b & 0 & -a \\ 0 & c & 1 & d \end{pmatrix} \in \text{Gr}(2, 4)$$

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$$\begin{array}{llll} \Delta_{12}(C) = c & \Delta_{23}(C) = b & \Delta_{34}(C) = a & \Delta_{14}(C) = d \\ \Delta_{13}(C) = 1 & & \Delta_{24}(C) = ac + bd & \end{array}$$

Dimer model and positive Grassmannian

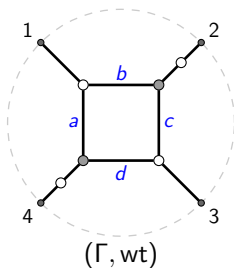
- $\text{Gr}(k, n) := \{C \subset \mathbb{R}^n \mid \dim C = k\} = \{C \in \text{Mat}_{k \times n}^{\text{rank}=k}(\mathbb{R})\} / \text{GL}_k(\mathbb{R})$ [$\dim = k(n - k)$].
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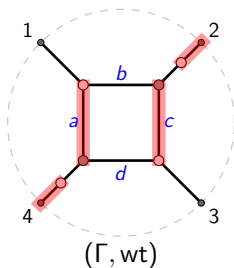


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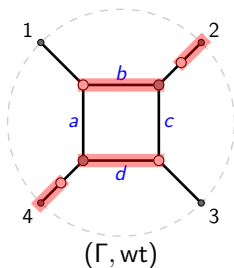


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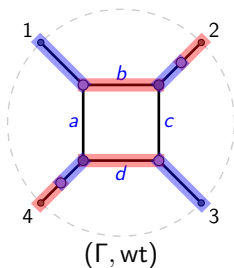
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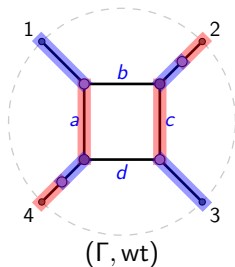
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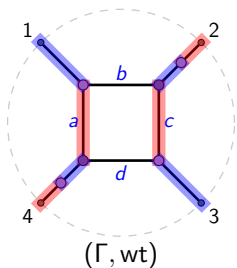
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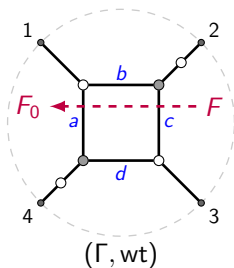
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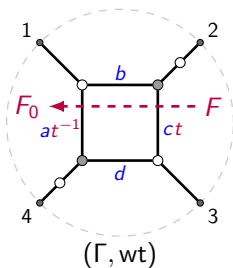
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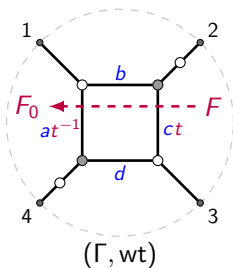
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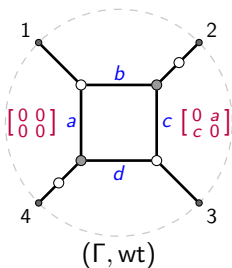
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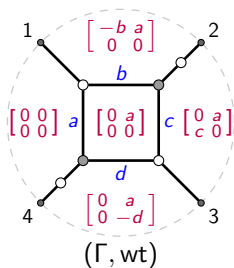
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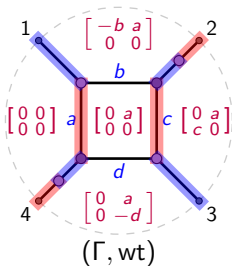
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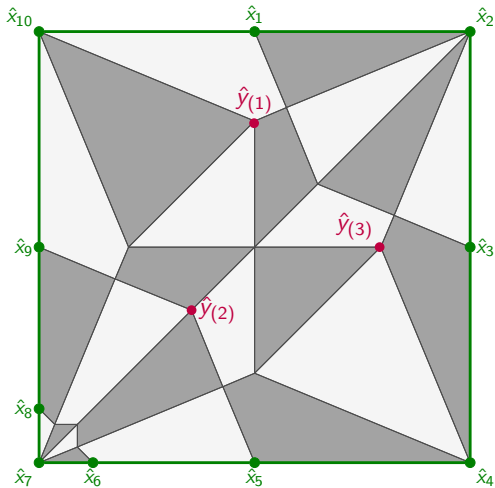
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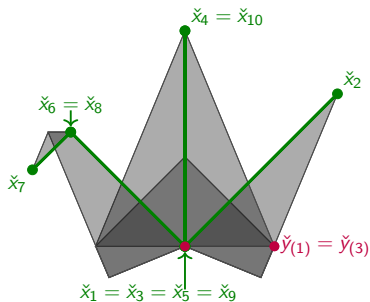
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- **M-positivity:** $\text{rank}(C'_{F_i}(1) - C'_{F_j}(1)) \geq 2 \iff F_i$ and F_j are **2-separated**
 (separated by at least 2 double-dimer paths).

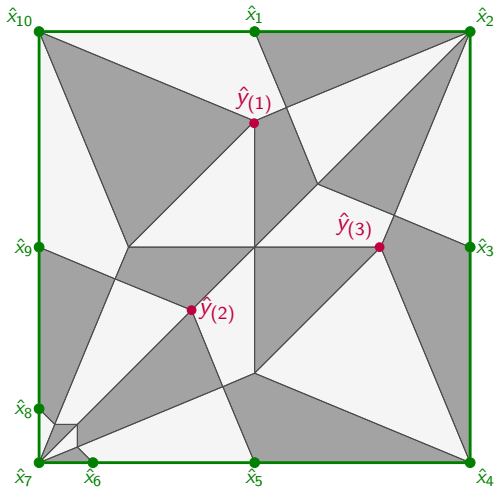


“Space” ($\mathbb{R}^2$)

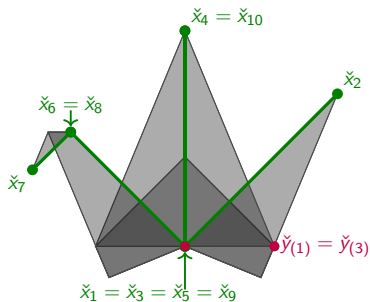


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- OCP + folding: embed $\Gamma^* \rightarrow \mathbb{R}^{2,2}$ with Minkowski norm $|x|^2 = |\hat{x}|^2 - |\check{x}|^2$.

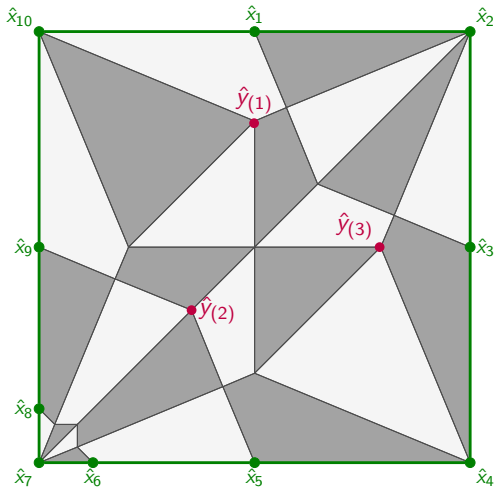


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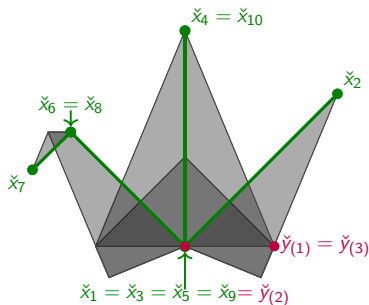


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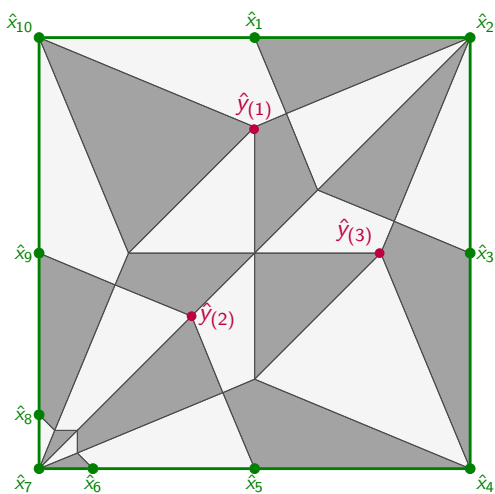


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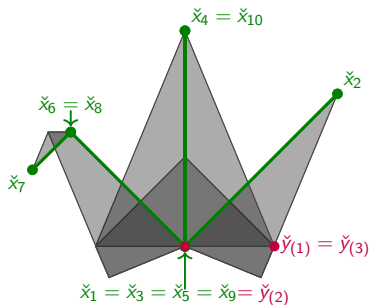


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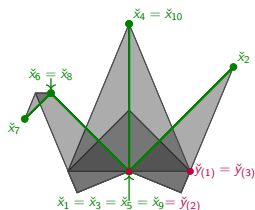
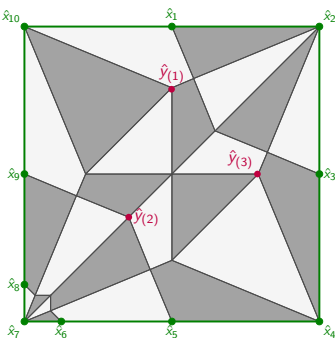


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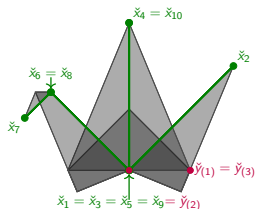
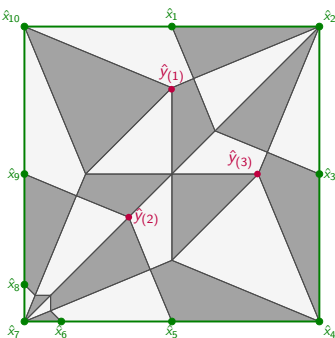


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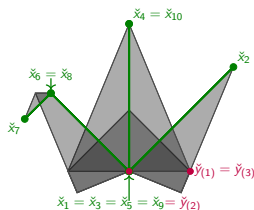
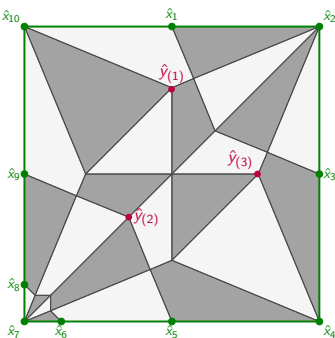
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 - Every OCP is a linear projection of a Kenyon–Smirnov primitive.
- [Kenyon–Lam–Ramassamy–Russkikh '18], [Chelkak–Laslier–Russkikh '21]



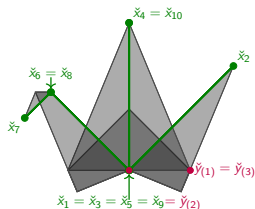
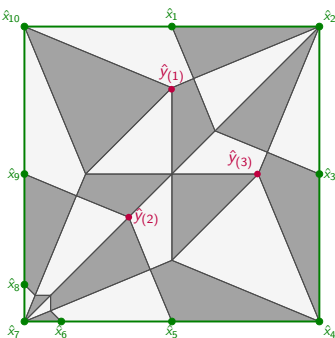
- OCP + folding: embed $\Gamma^* \rightarrow \mathbb{R}^{2,2}$ with Minkowski norm $|x|^2 = |\hat{x}|^2 - |\check{x}|^2$.
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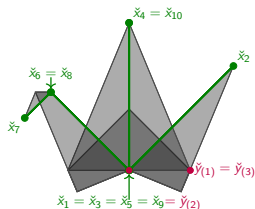
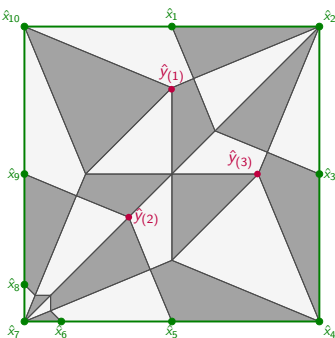
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Theorem (Origami–Amplituhedron Correspondence [G.'24])

A Kenyon–Smirnov primitive gives rise to an *embedded OCP* $\iff$

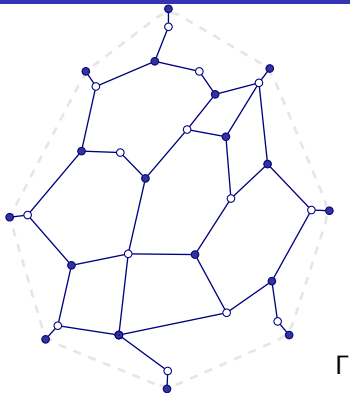


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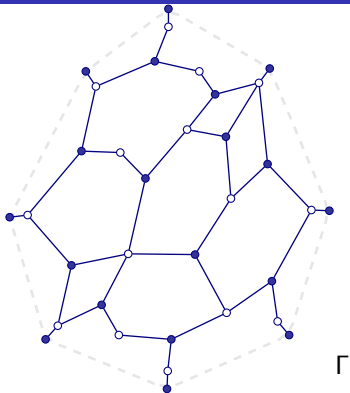
A Kenyon–Smirnov primitive gives rise to an embedded OCP $\iff$ its $\mathbb{R}^{2,2}$ -boundary polygon belongs to **momentum amplituhedron** $\mathcal{M}_{k,n}^{(L=0)}$.

T-duality for planar bipartite graphs



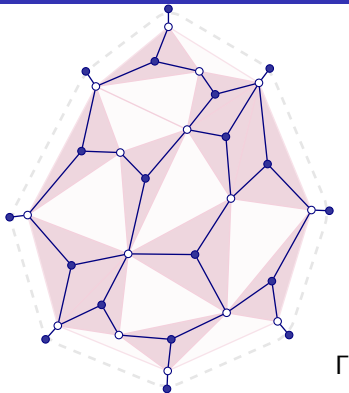
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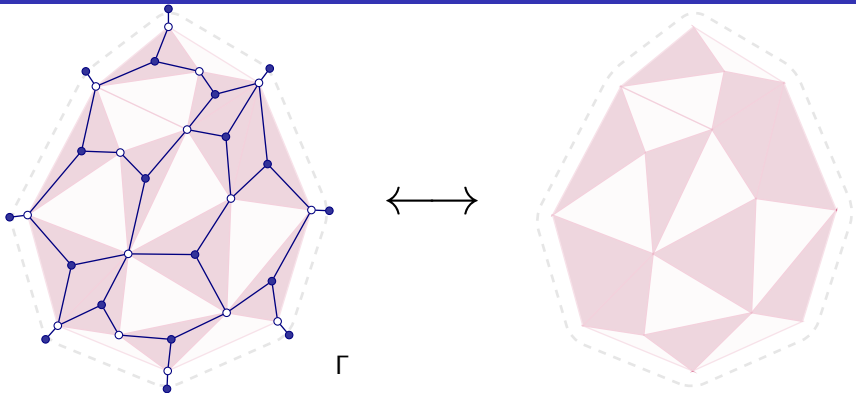
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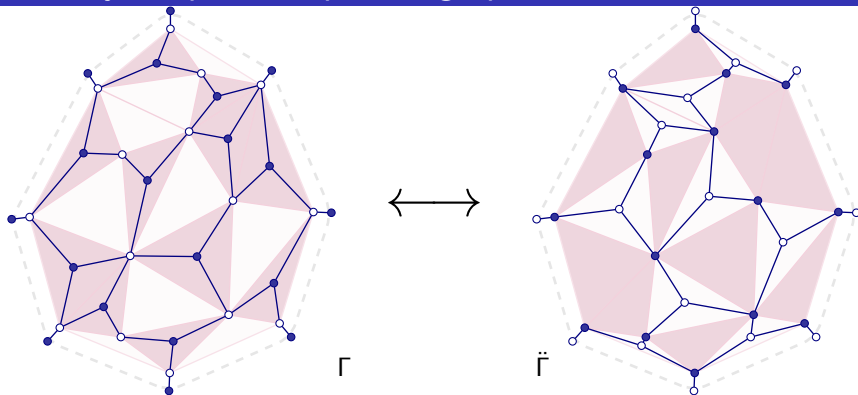
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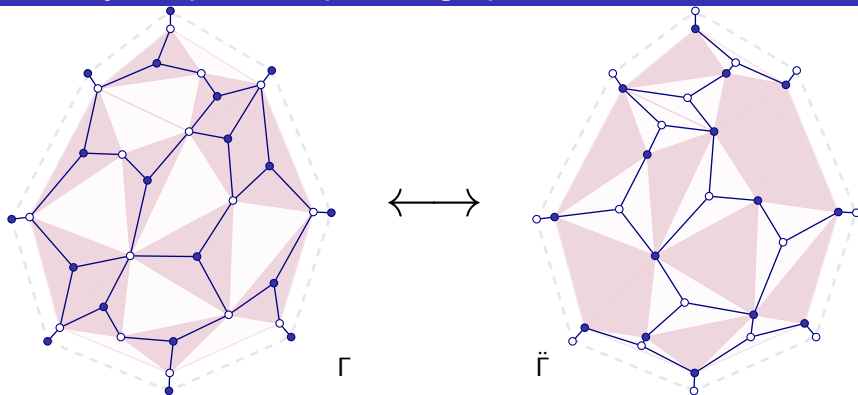
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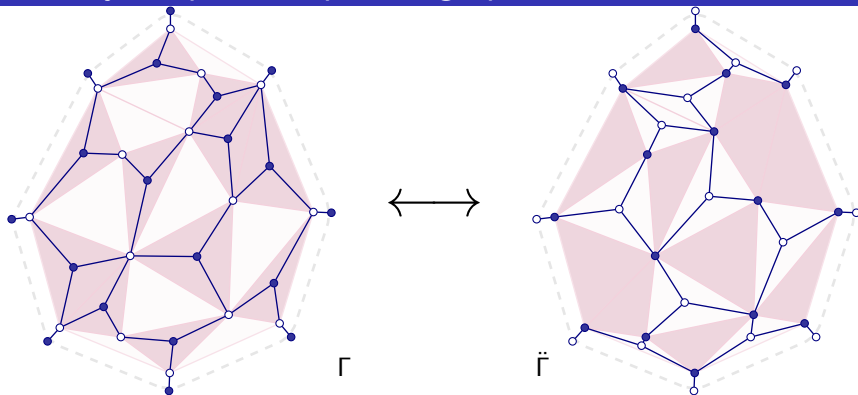
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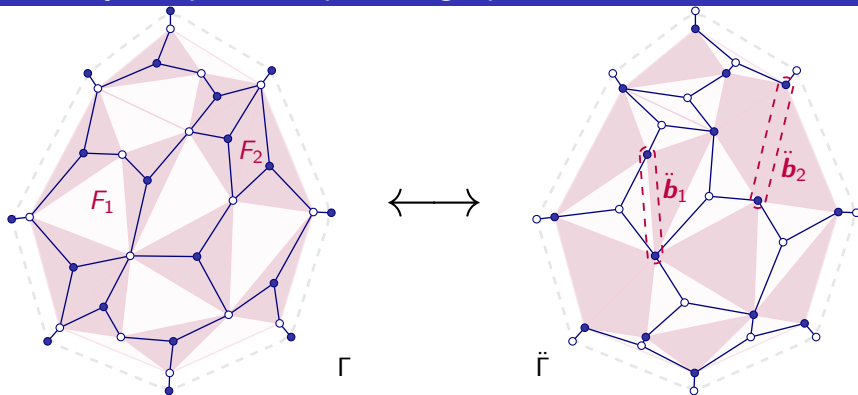
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T-duality for planar bipartite graphs

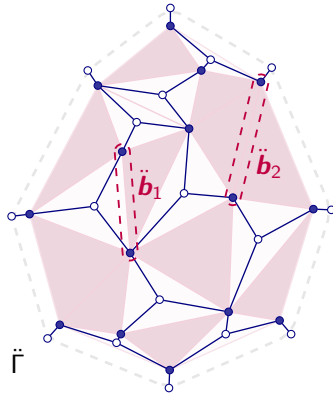
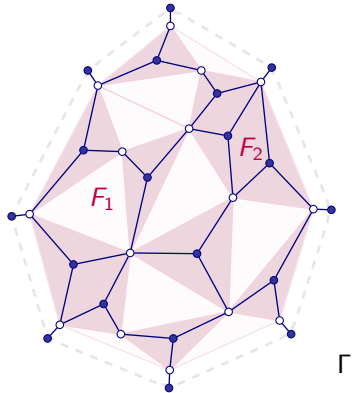


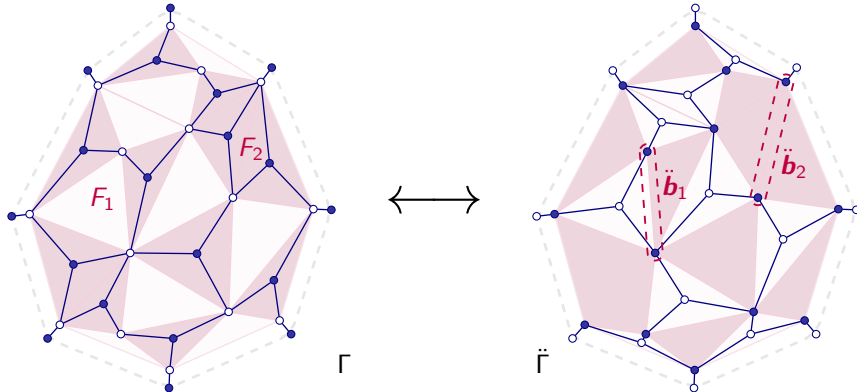
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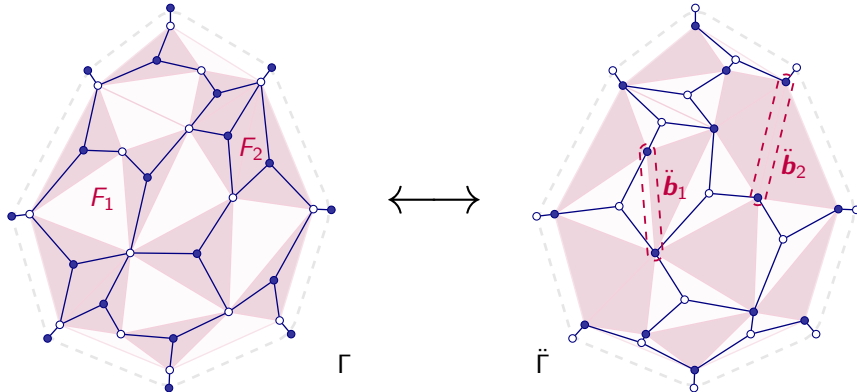
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- **Faces** of Γ correspond to **pairs of black vertices** in $\tilde{\Gamma}$.





Theorem (T-duality [G.'26])

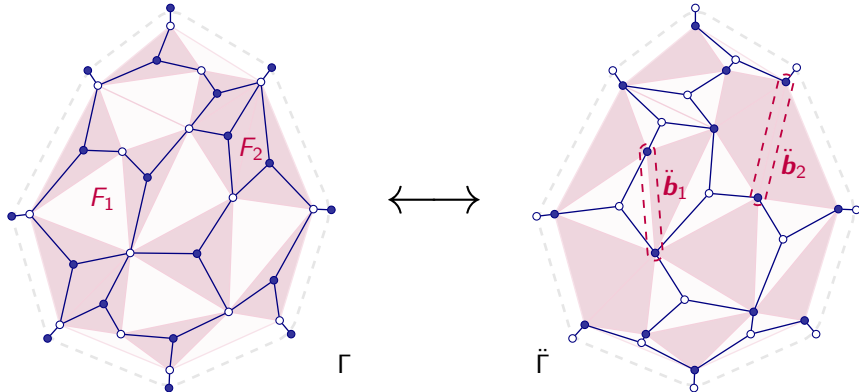
For each face F_i of Γ , let $\tilde{b}_i$ consist of any two black vertices incident to F_i .



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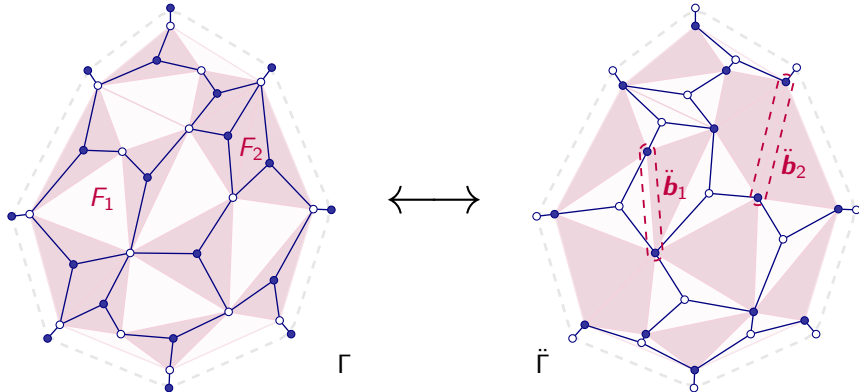
- Boundary measurements of (Γ, wt) and $(\tilde{\Gamma}, \tilde{\text{wt}})$ are related by the **magic projector** $\text{Gr}_{\geq 0}(k, n) \rightarrow \text{Gr}_{\geq 0}(k - 2, n)$ of [Arkani-Hamed–Cachazo–Cheung '10].



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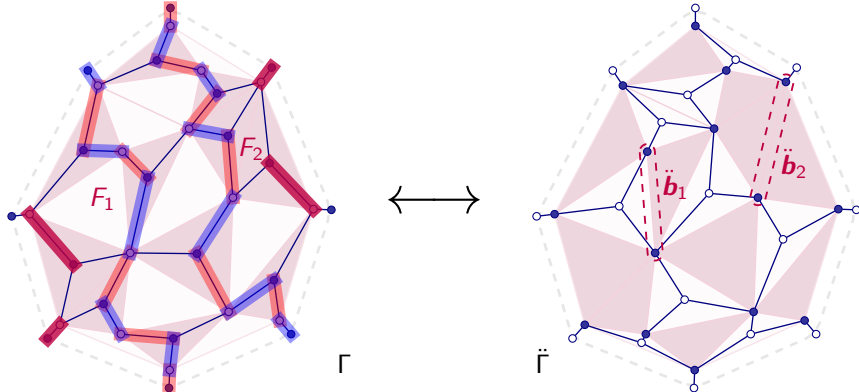
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- F_i and F_j are 2-separated $\iff \tilde{\Gamma} \setminus (\tilde{\mathbf{b}}_i \cup \tilde{\mathbf{b}}_j)$ admits a perfect matching

$\Updownarrow$
M-positivity for $\mathcal{M}_{k,n}^{(L)}$

$\Updownarrow$
T-dual M-positivity for $\mathcal{A}_{k-2,n}^{(L)}$



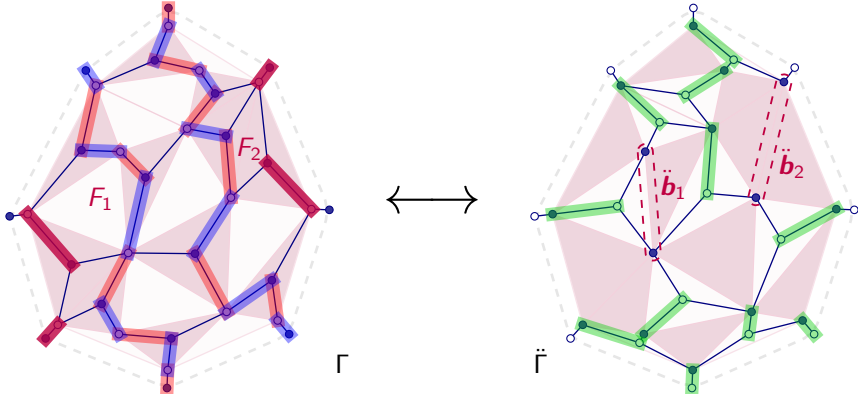
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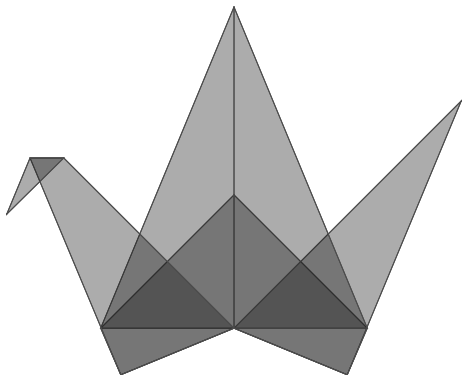
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Thanks!