Amplituhedra and Origami

Pavel Galashin (Cornell)

KOI Combinatorics Lectures

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arXiv:2410.09574v2, arXiv:2606.03439

galashin.github.io/origami

Kawasaki's theorem

6 languages

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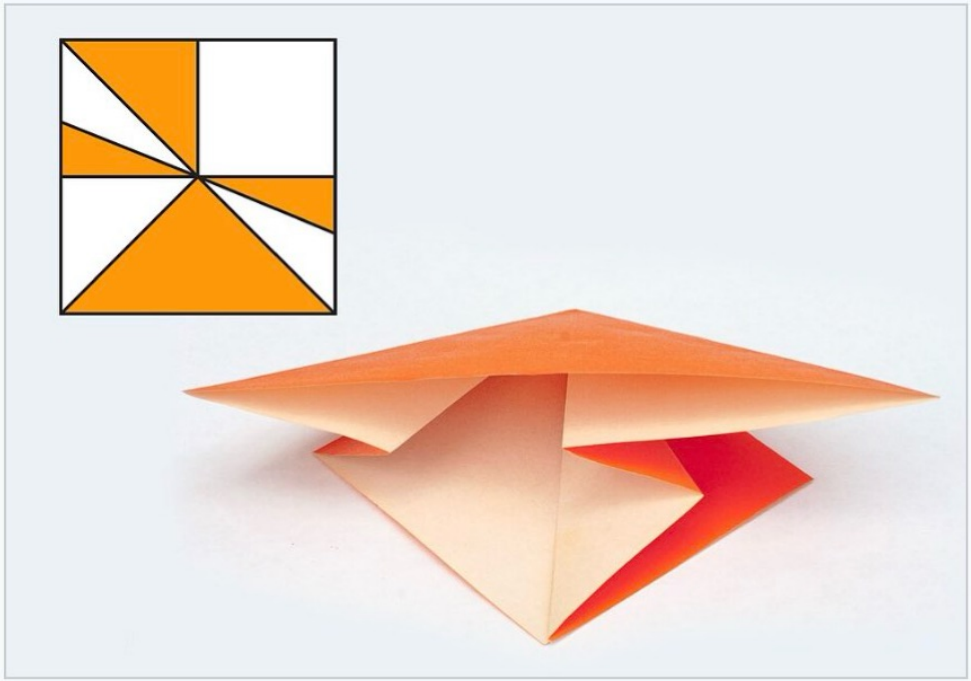
Kawasaki's theorem or **Kawasaki–Justin theorem** is a theorem in the mathematics of paper folding that describes the crease patterns with a single vertex that may be folded to form a flat figure. It states that the pattern is flat-foldable if and only if alternatingly adding and subtracting the angles of consecutive folds around the vertex gives an alternating sum of zero. Crease patterns with more than one vertex do not obey such a simple criterion, and are NP-hard to fold.



In this example, the alternating sum of angles (clockwise from the bottom) is 90° – 45° + 22.5° – 22.5° + 45° – 90° + 22.5° – 22.5° = 0°. Since it adds to zero, the crease pattern may be flat-folded.

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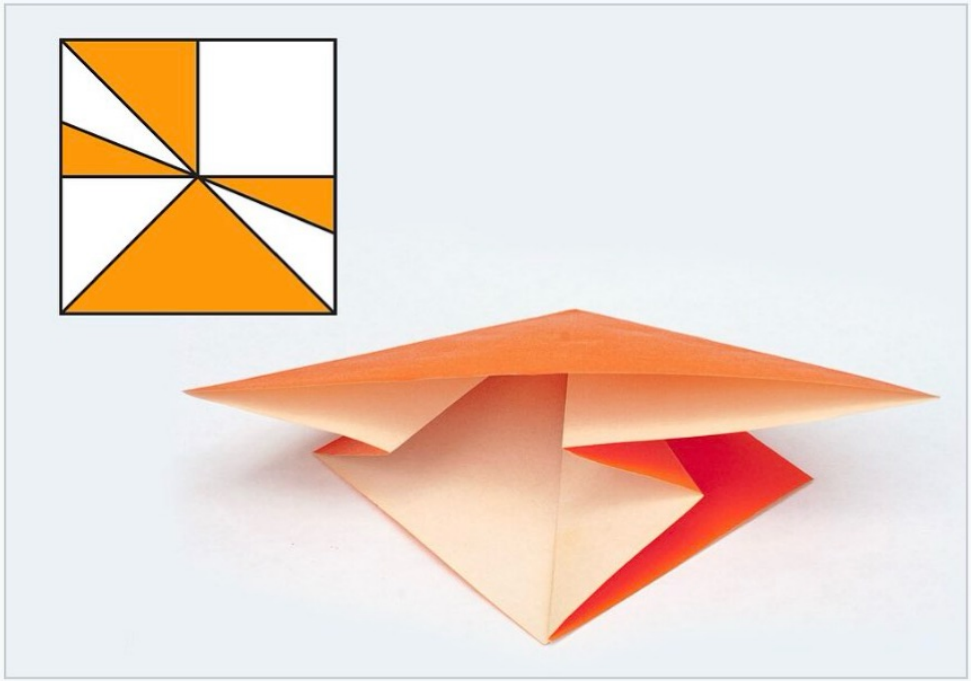

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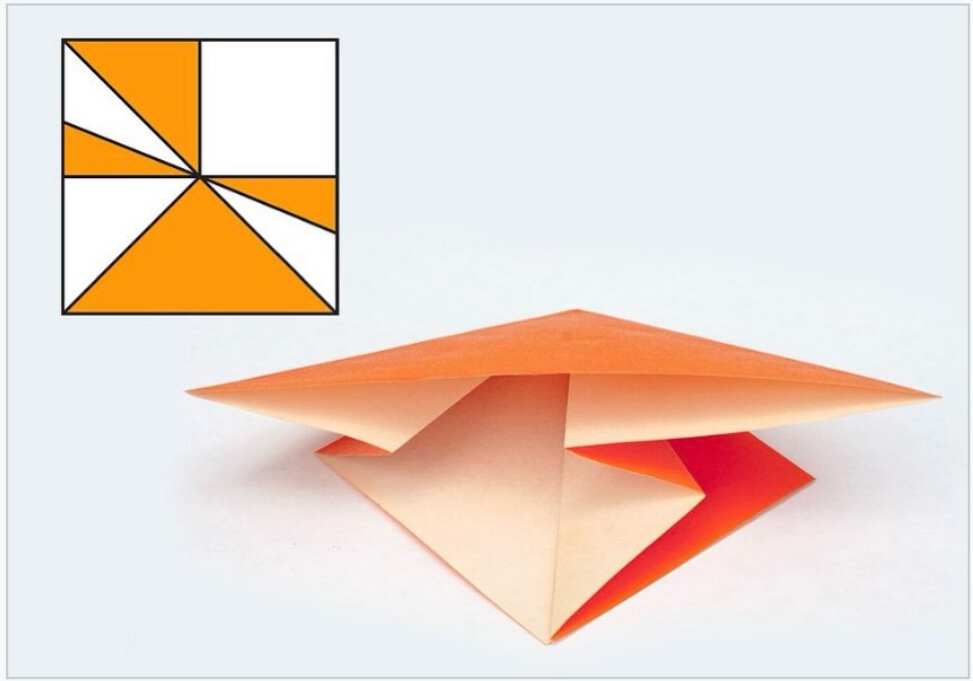

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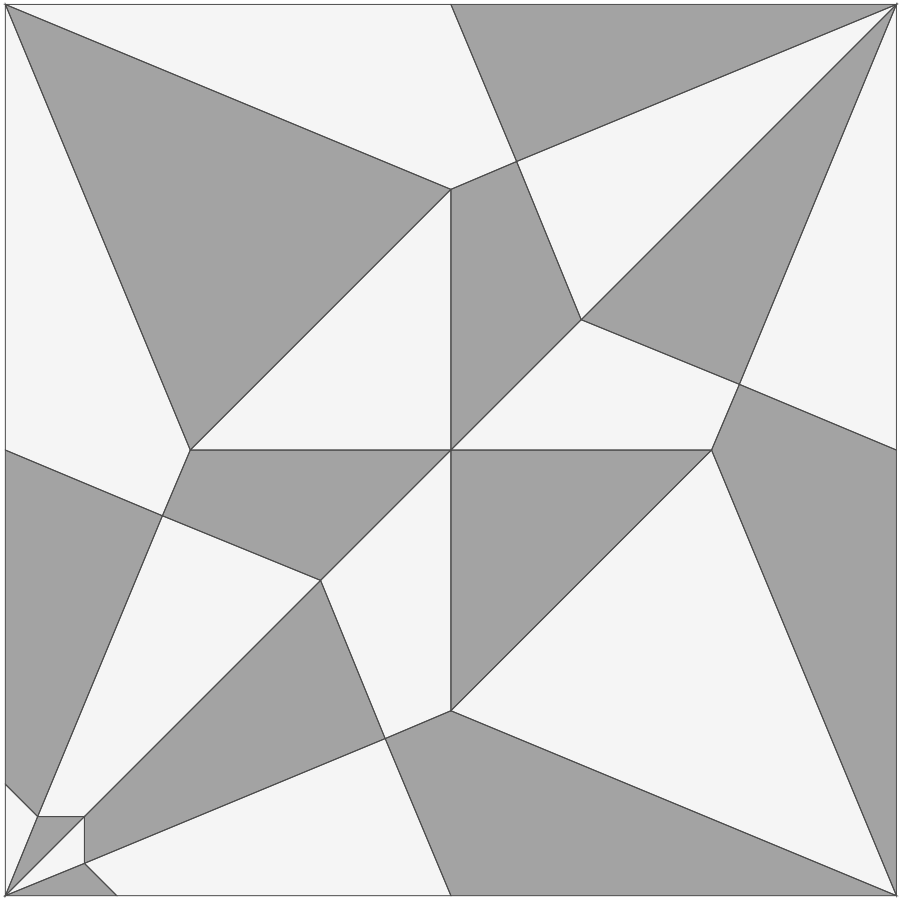
Origami crease patterns (OCP)



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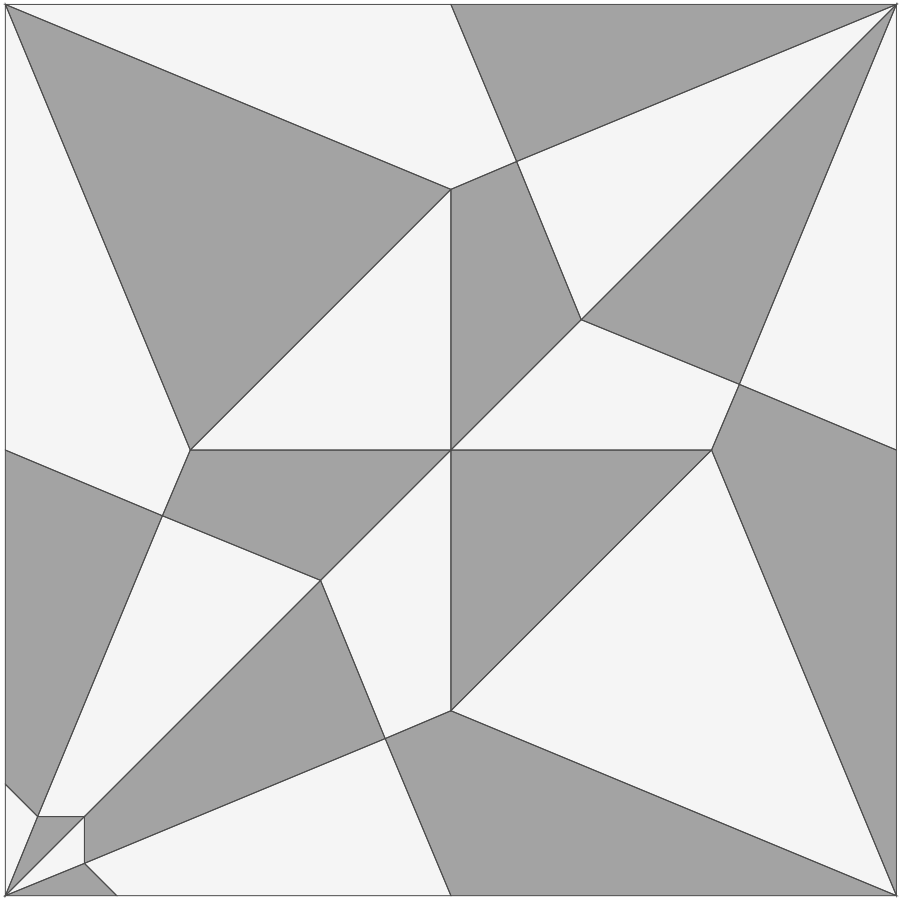

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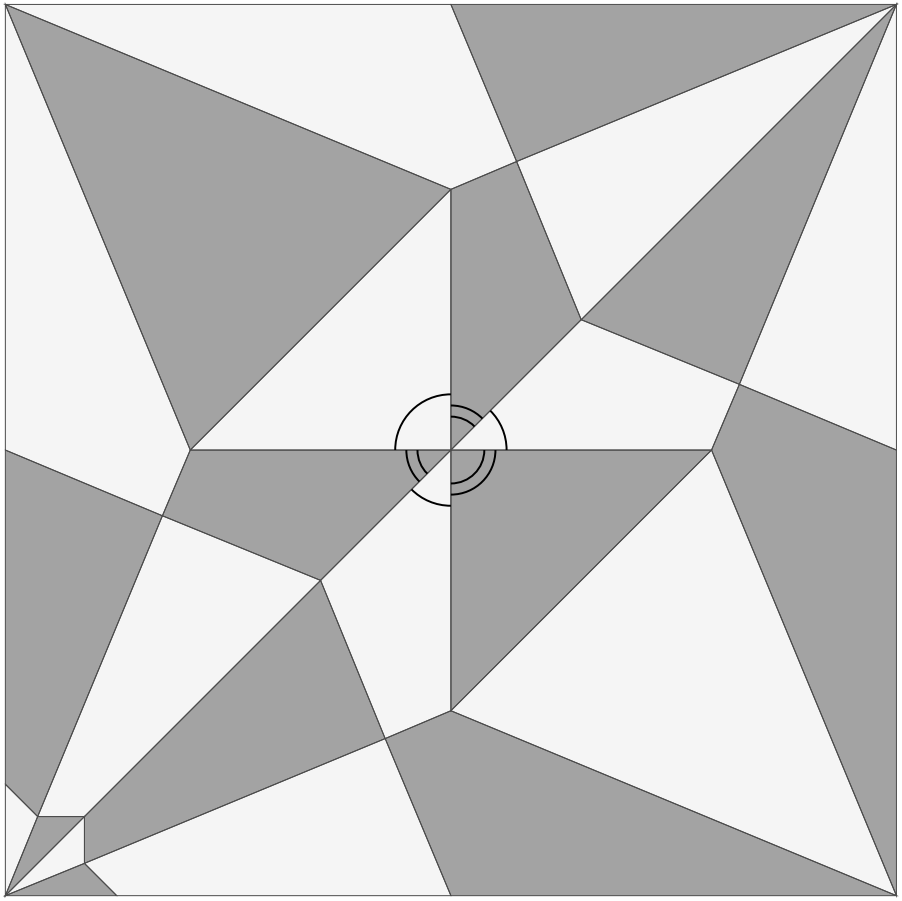

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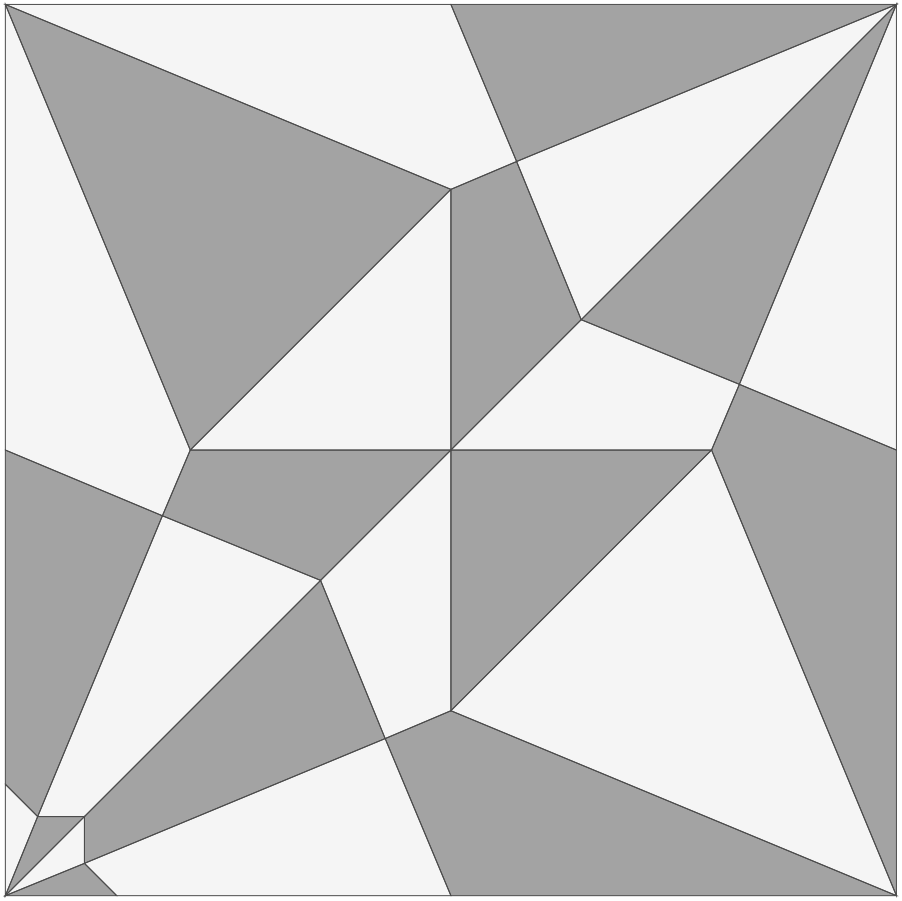

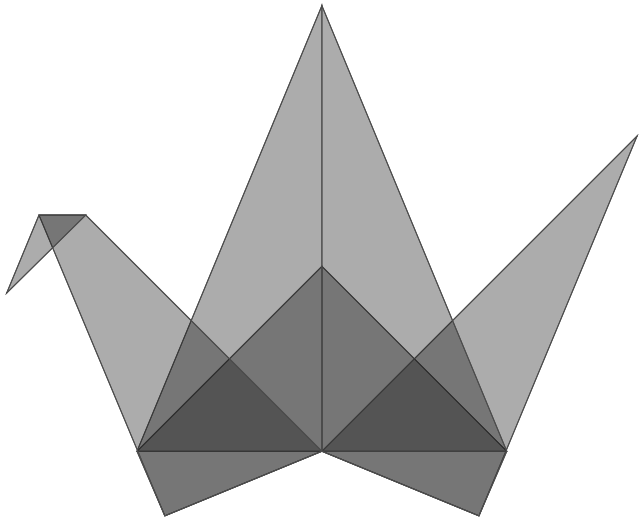

- Faces: convex polygons colored black and white;
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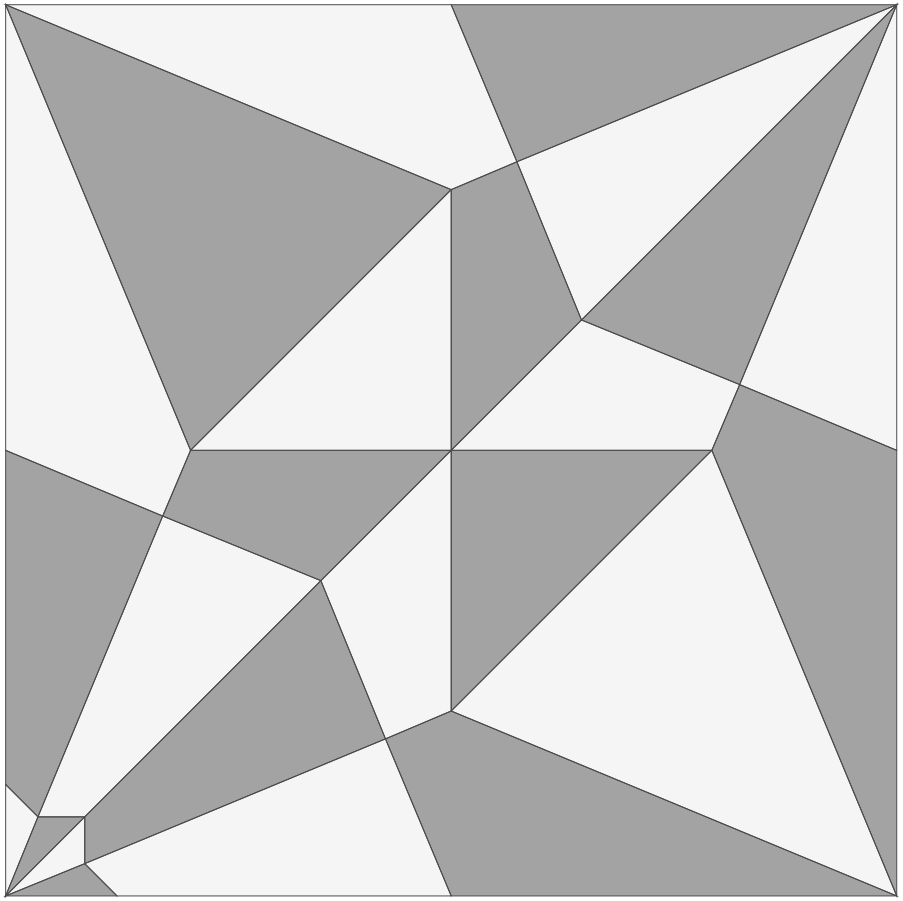
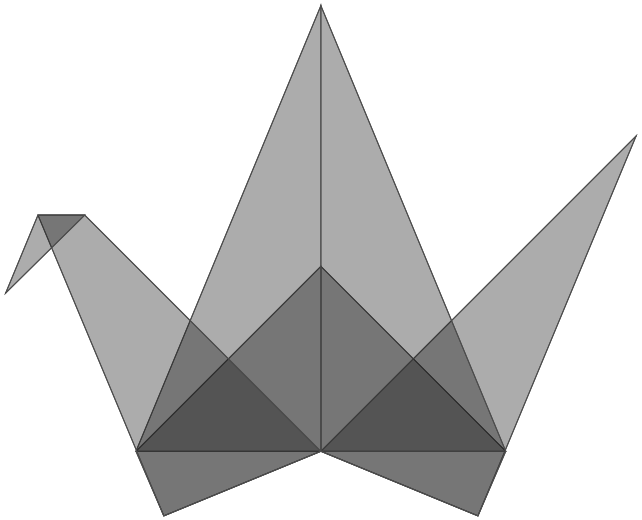
- Origami map: isometry on each face preserving/reversing the orientations of white/black faces.



"Space" ($\mathbb{R}^2$)



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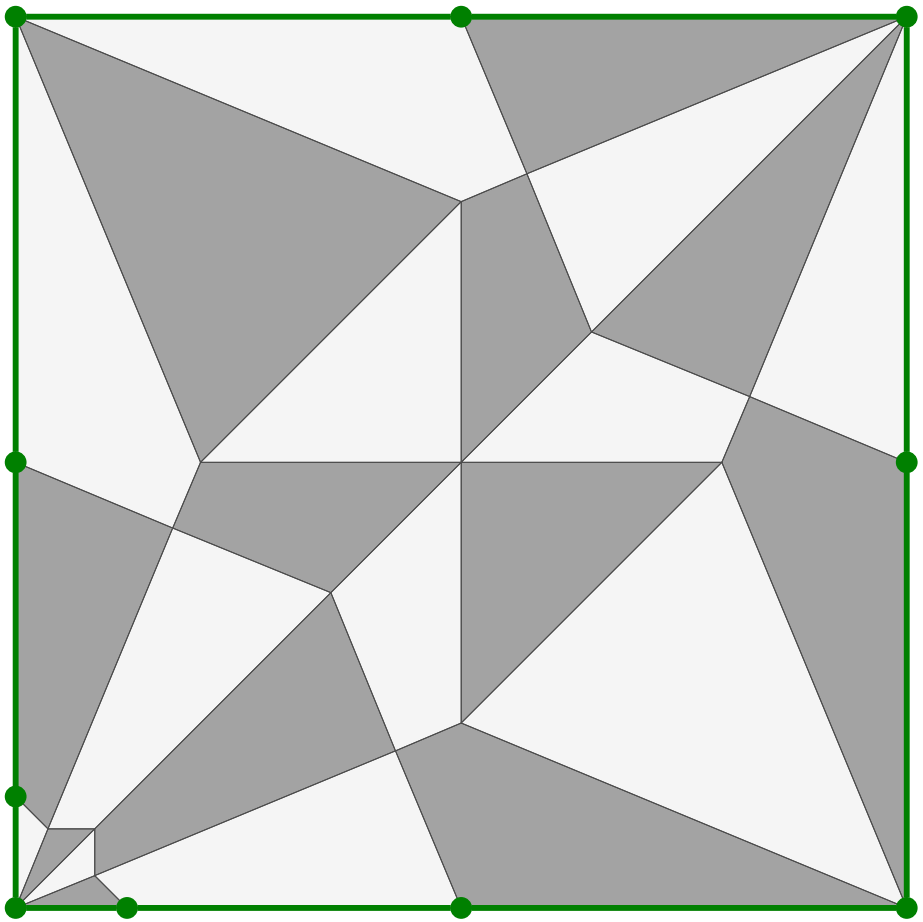

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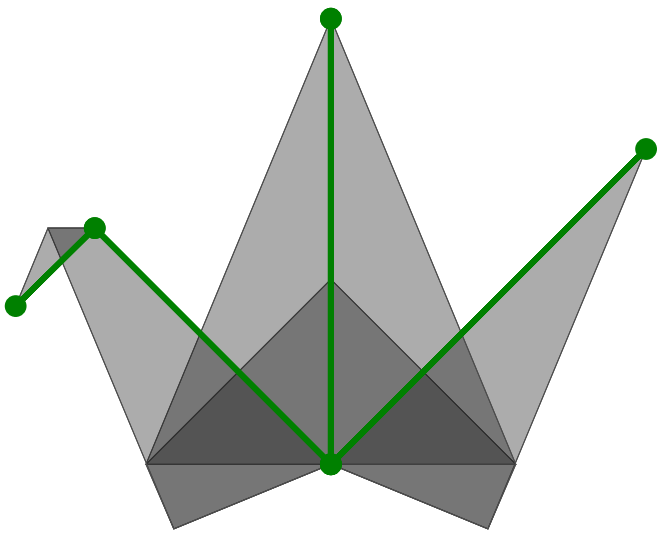

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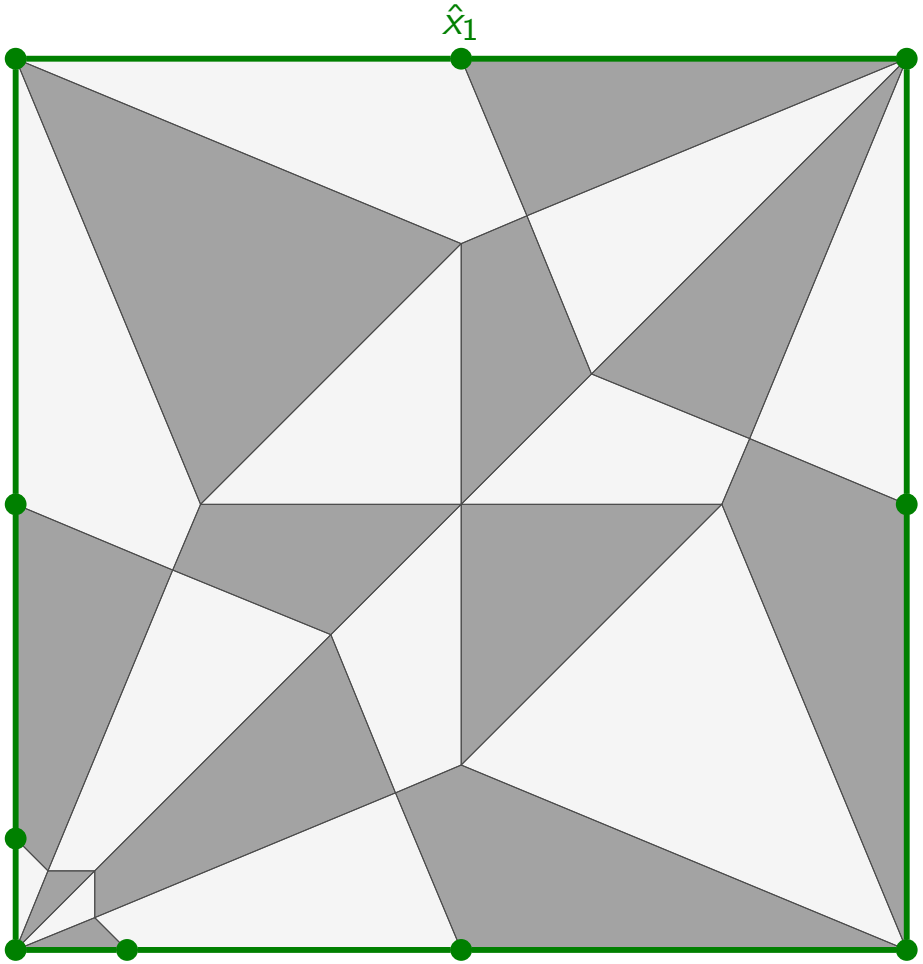

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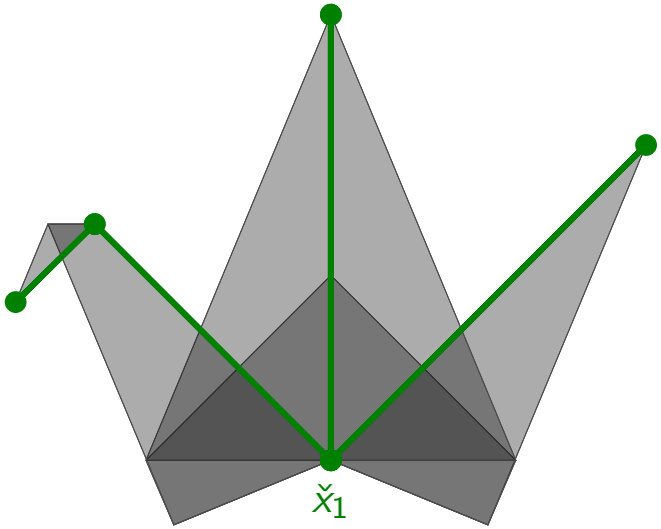

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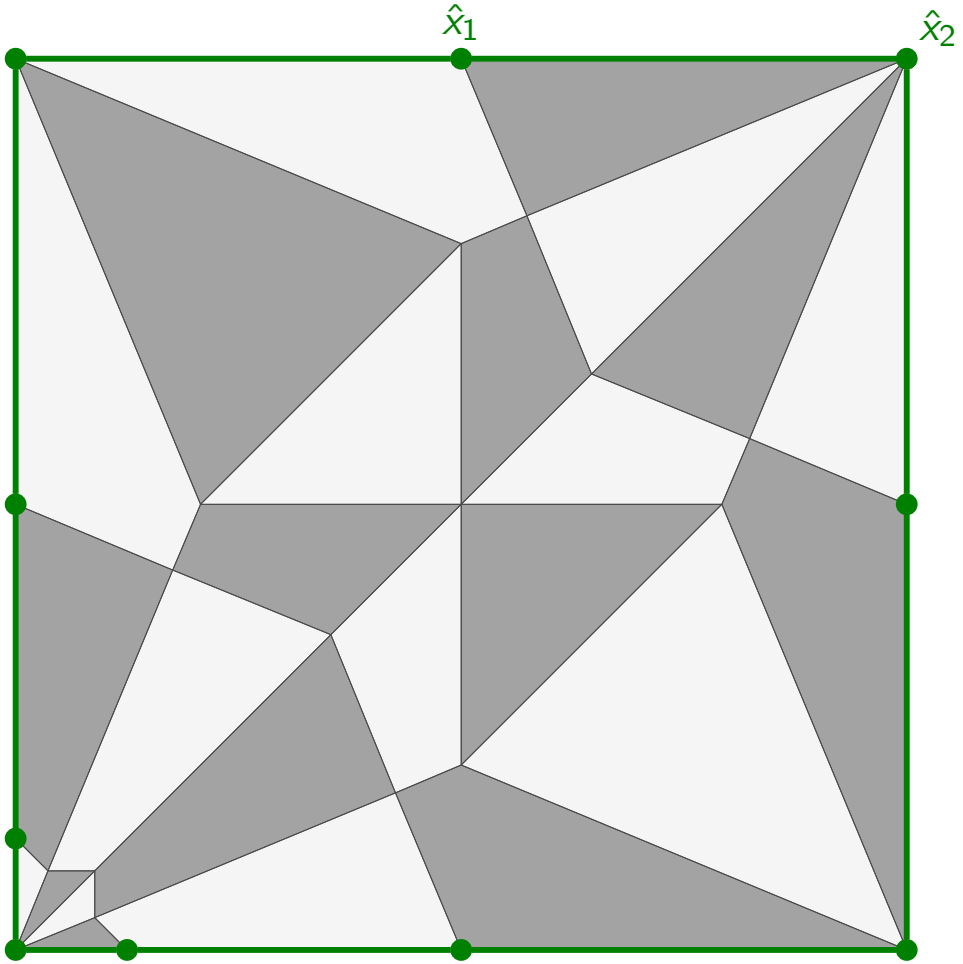

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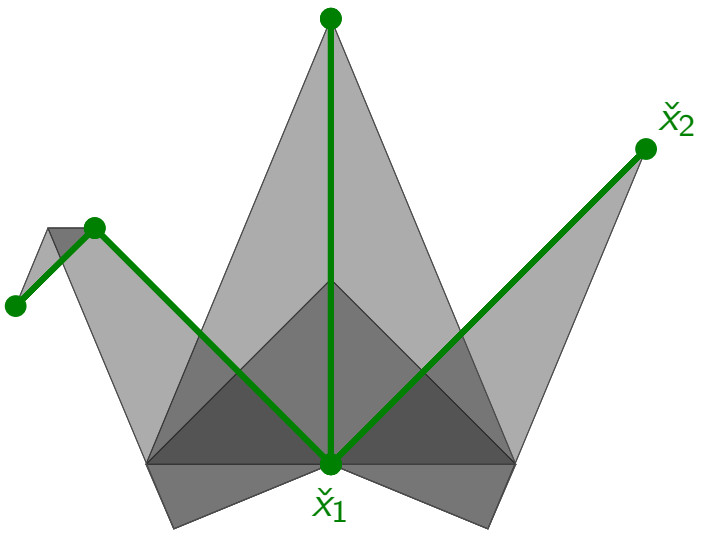

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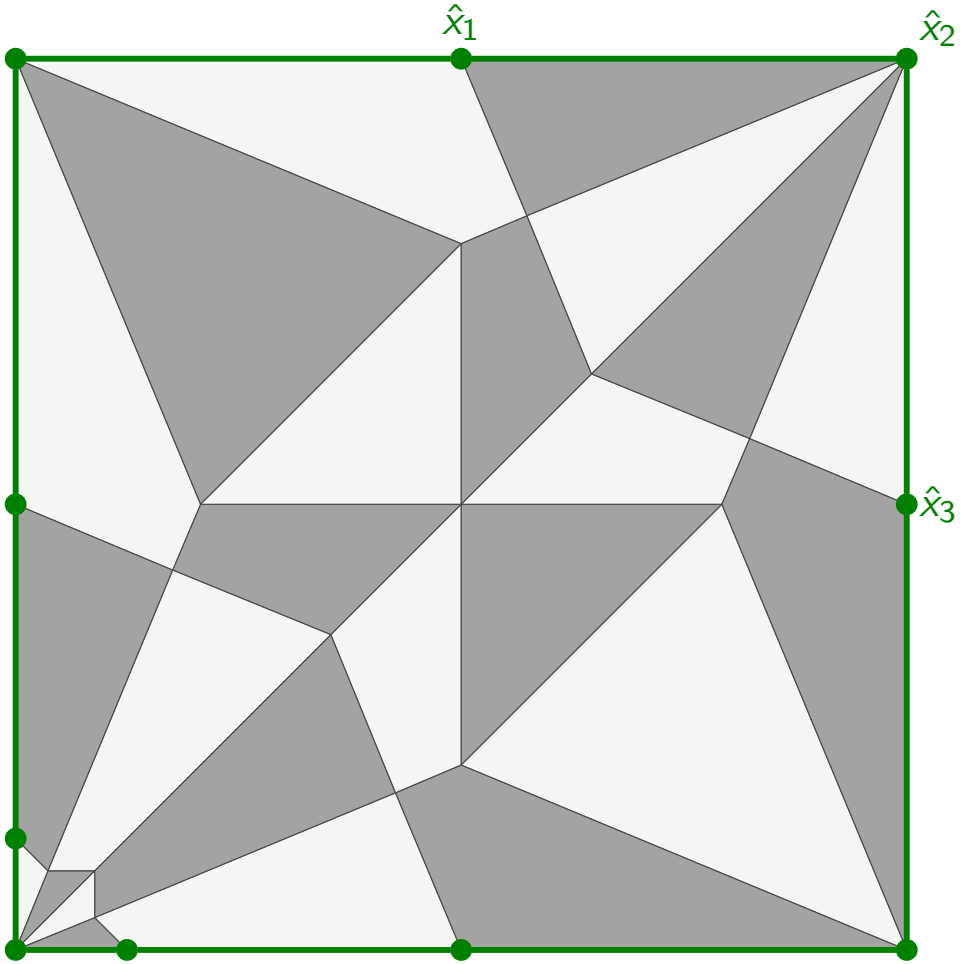

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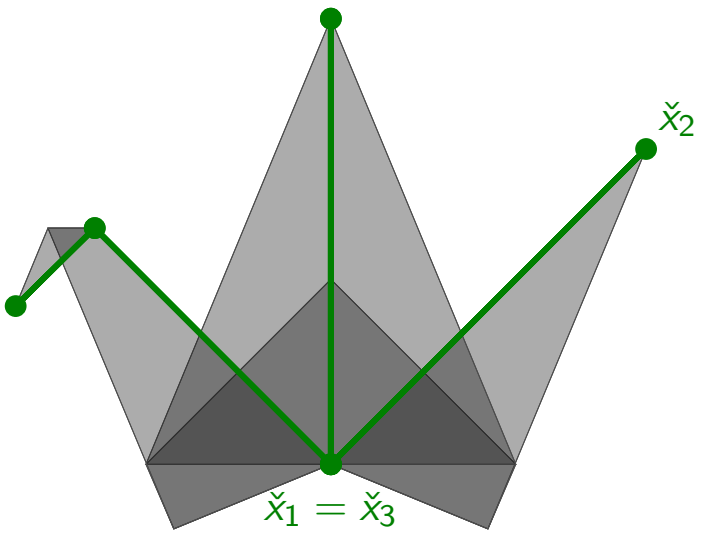

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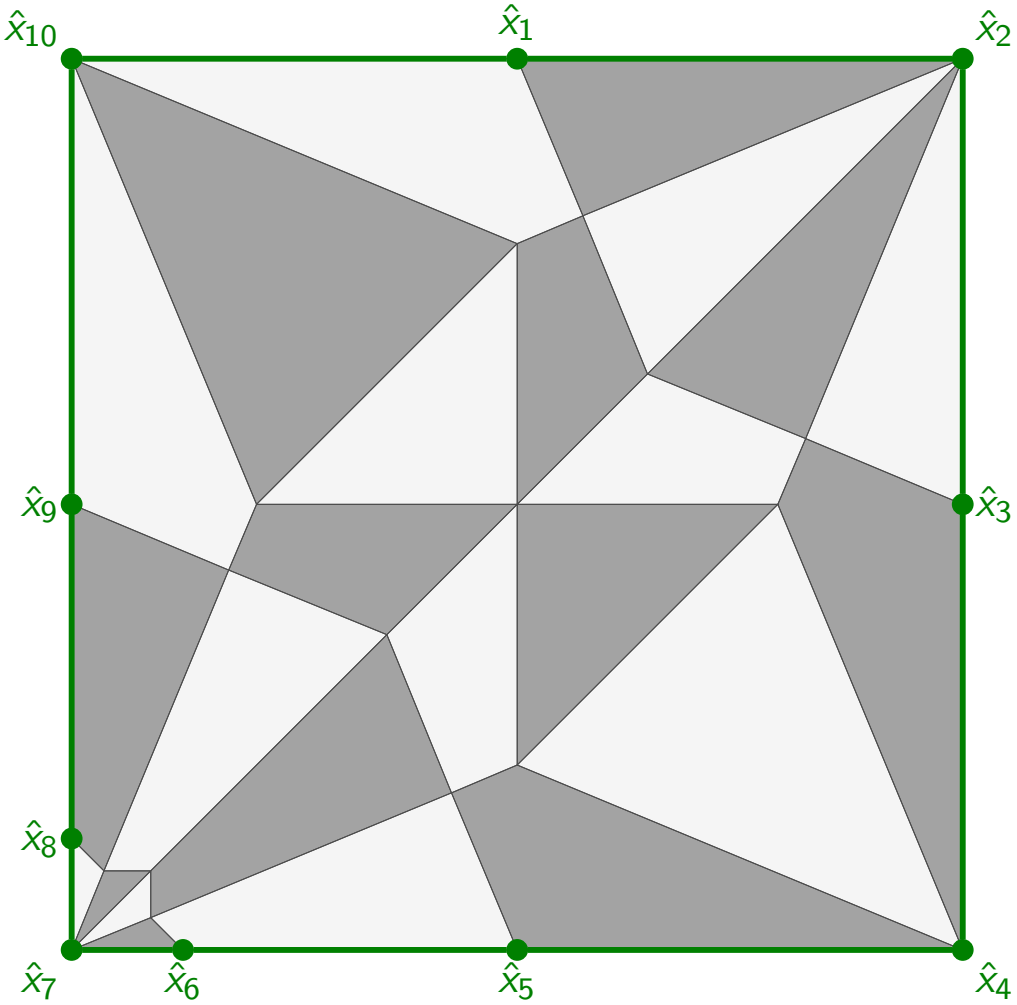

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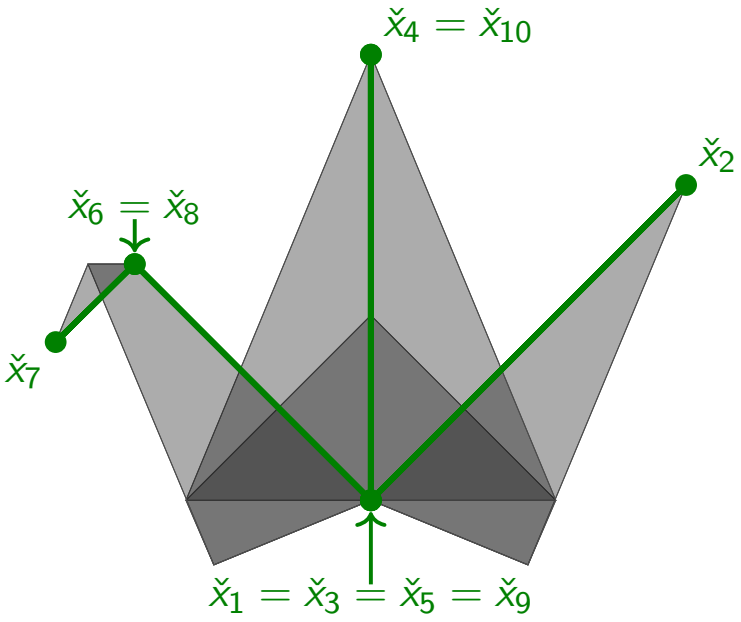

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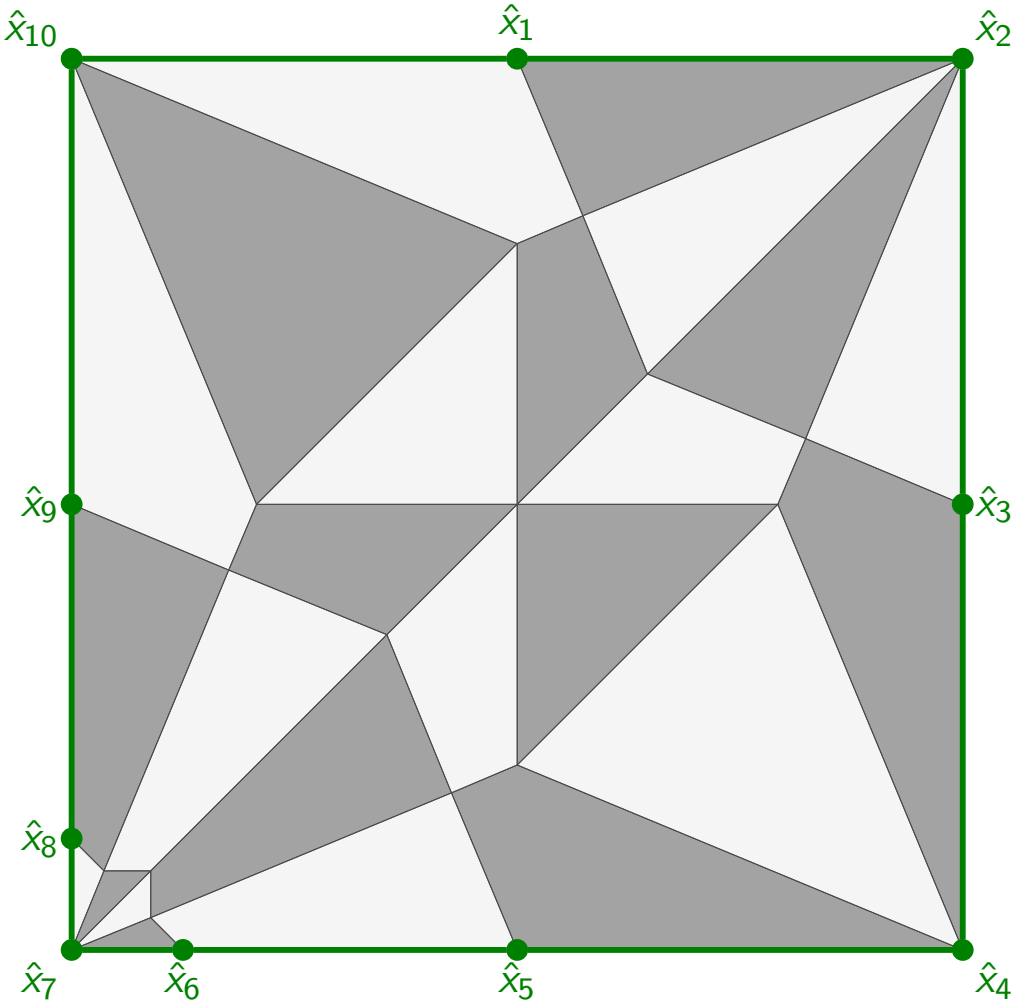

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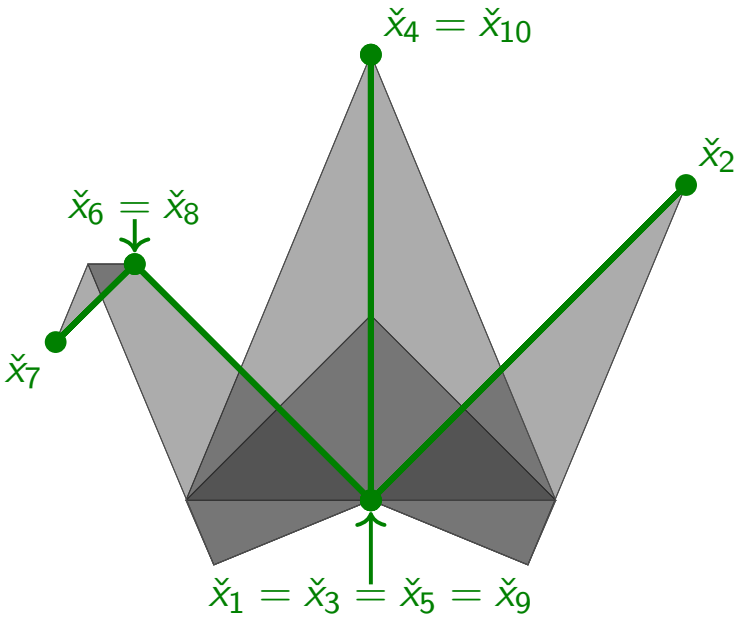

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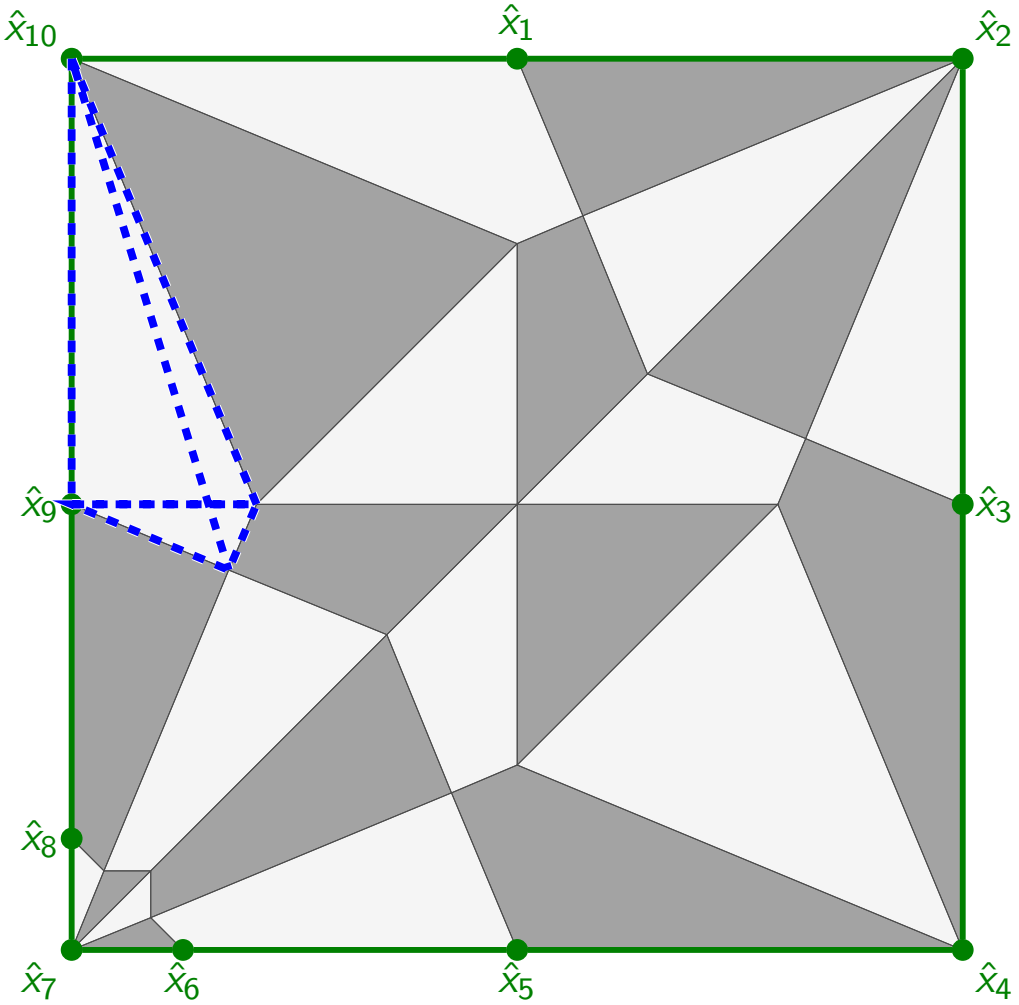

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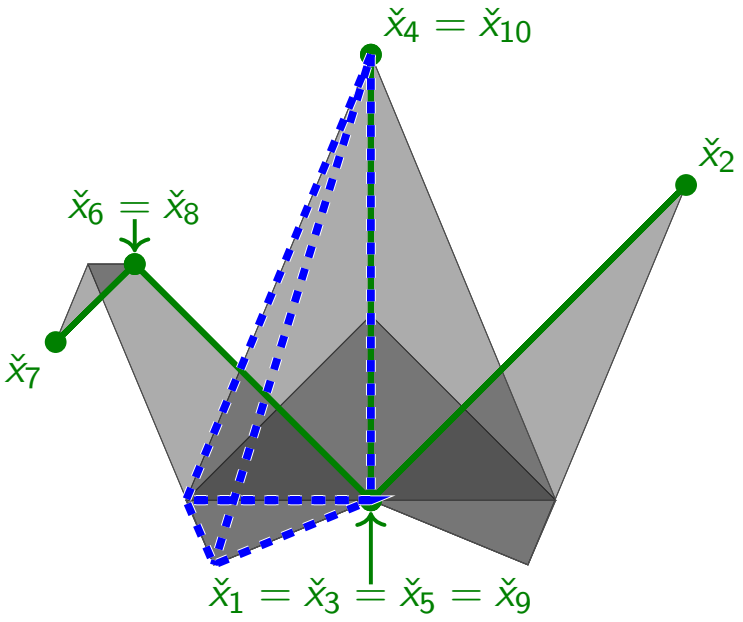

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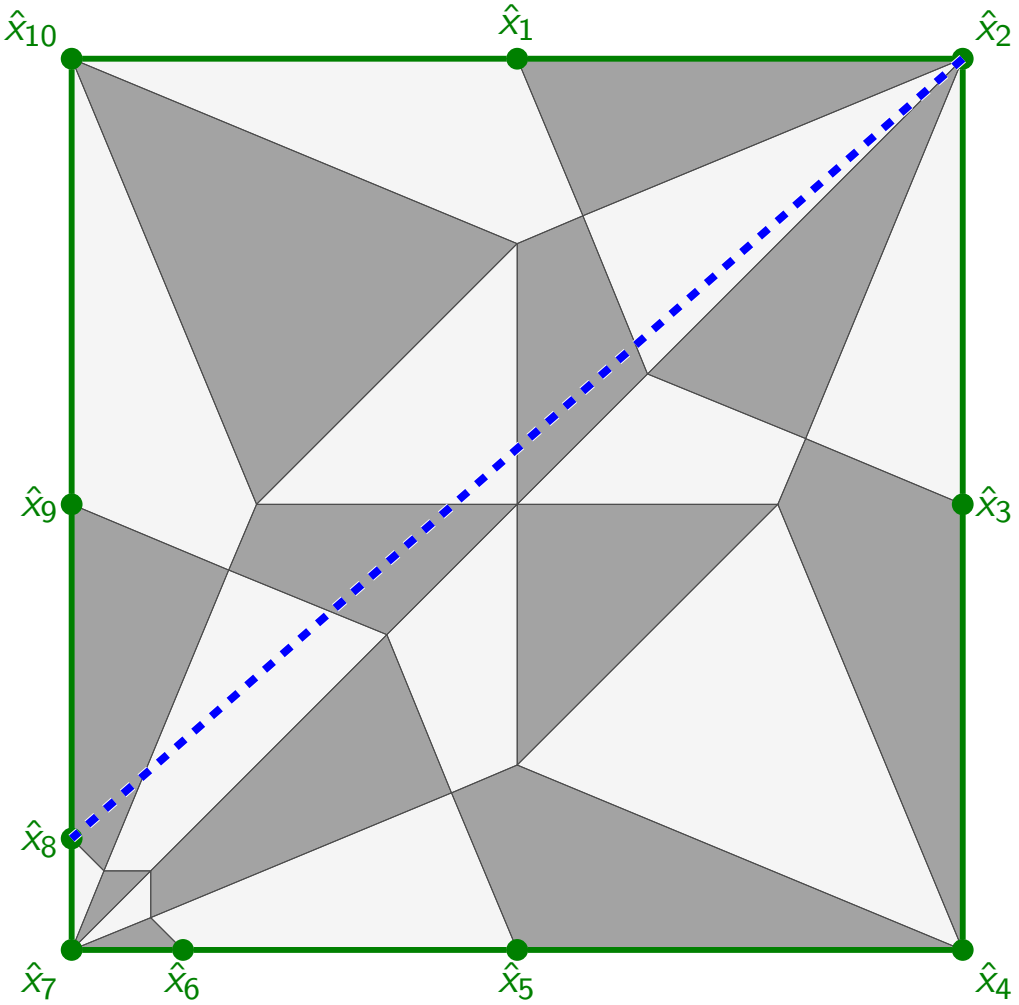

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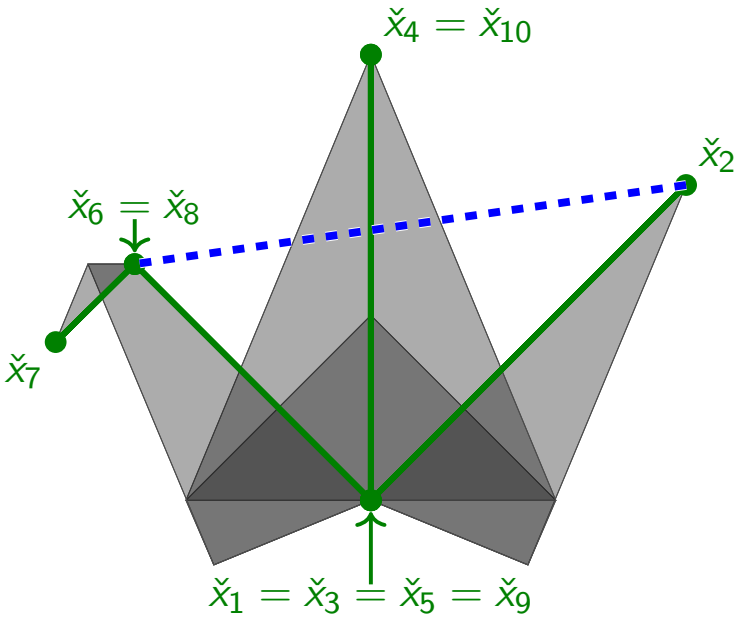

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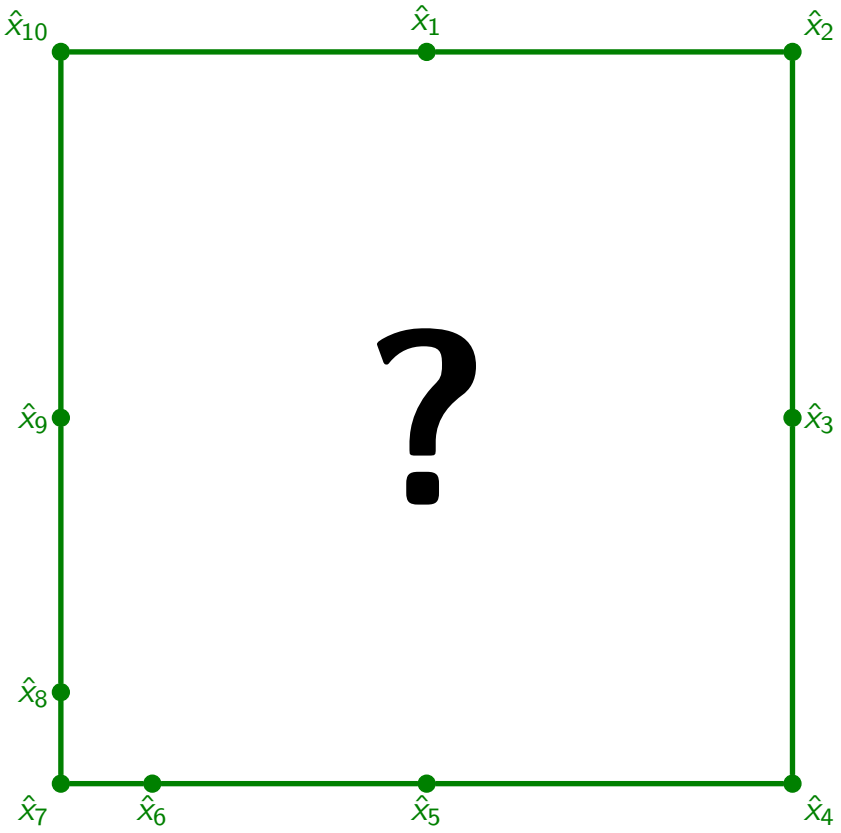

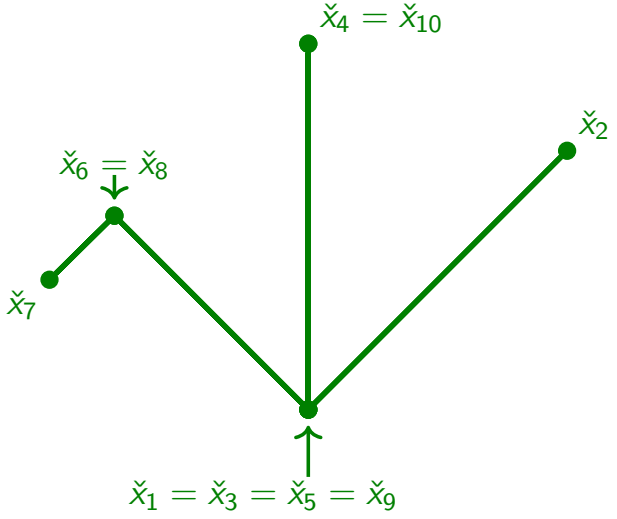

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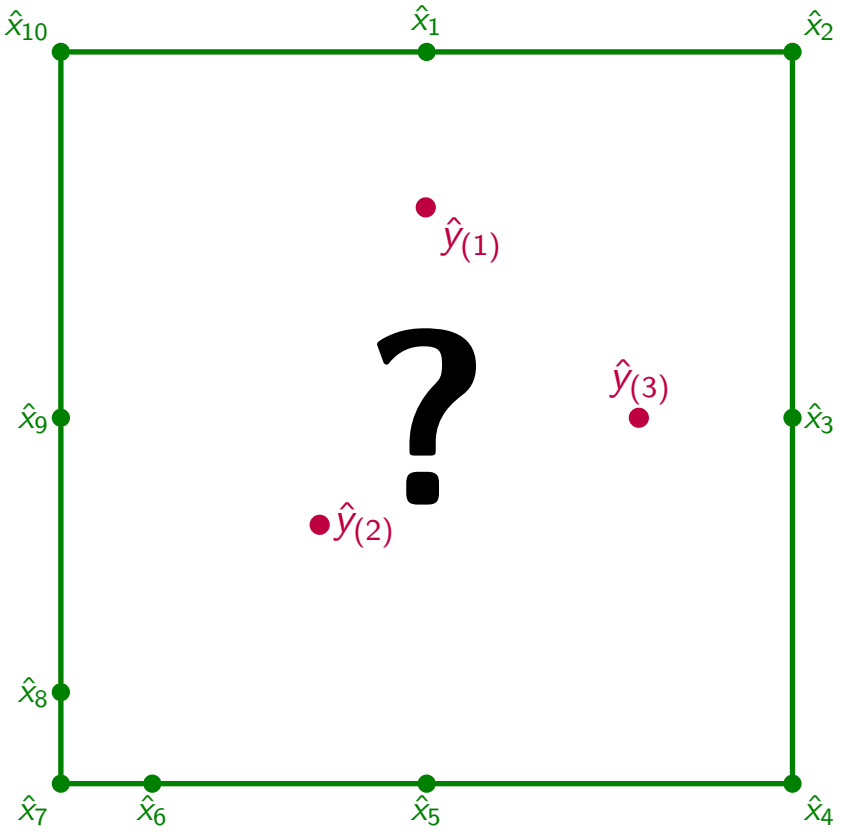
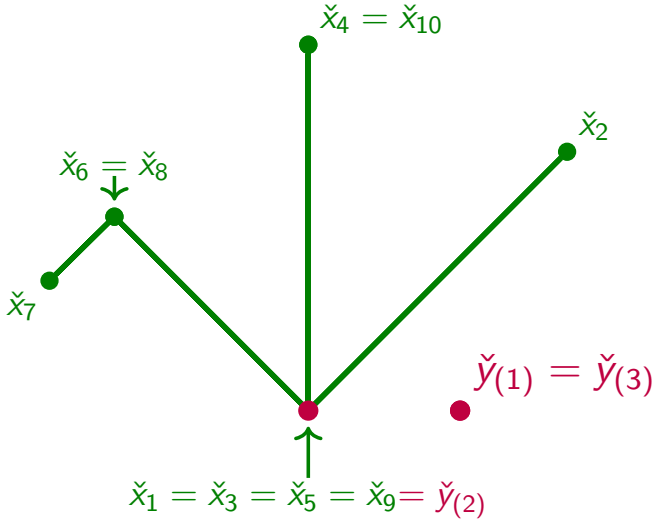
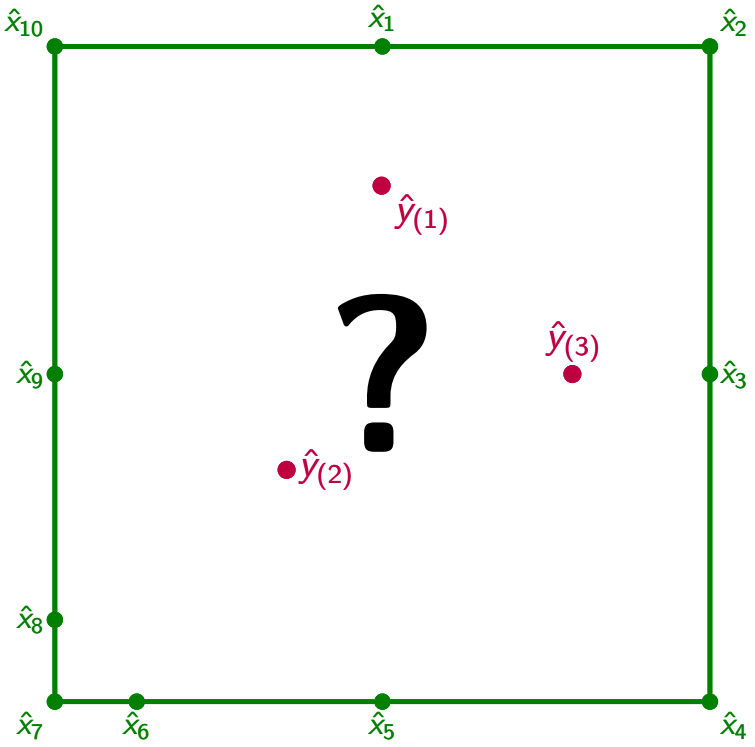
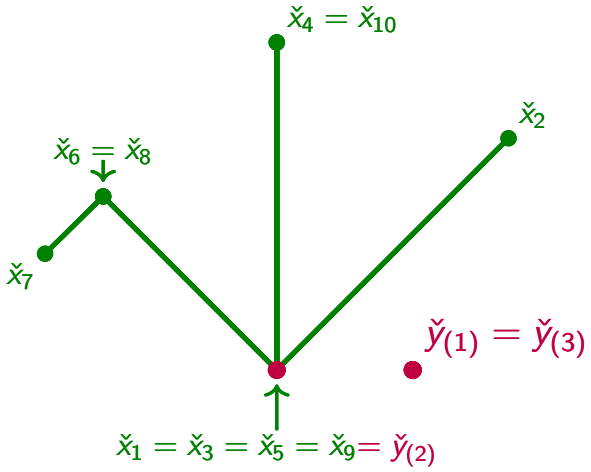
- ***L*-punctured origami reconstruction problem:** same as above but we additionally require the origami map to send $(\hat{y}_{(1)}, \ldots, \hat{y}_{(L)}) \mapsto (\check{y}_{(1)}, \ldots, \check{y}_{(L)})$.





L-punctured origami reconstruction problem: Find an OCP with boundary $(\hat{x}_1, \ldots, \hat{x}_n)$ such that the origami map sends $\hat{x}_i \mapsto \check{x}_i$ and $\hat{y}_{(j)} \mapsto \check{y}_{(j)}$ for all i, ρ.





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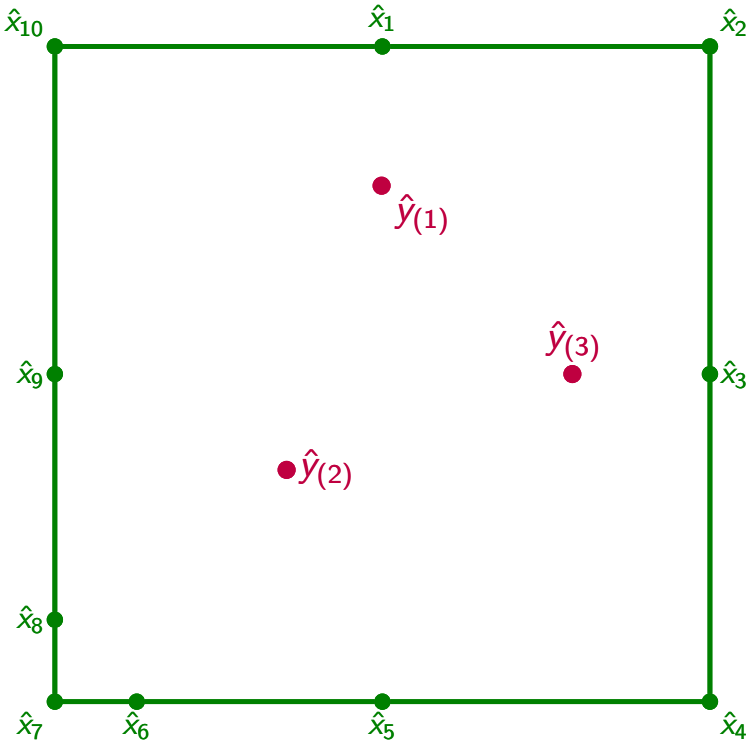

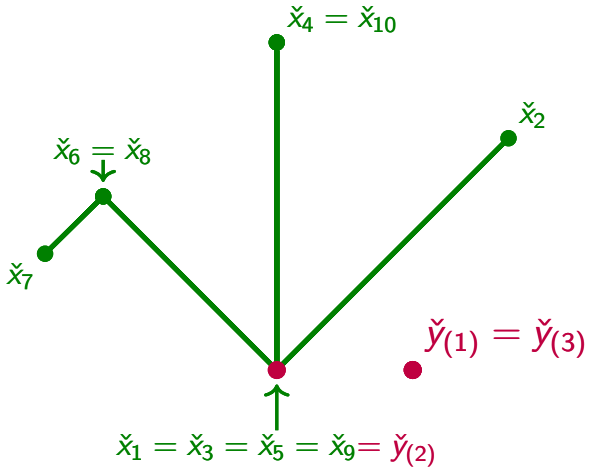

Origami Reconstruction Algorithm

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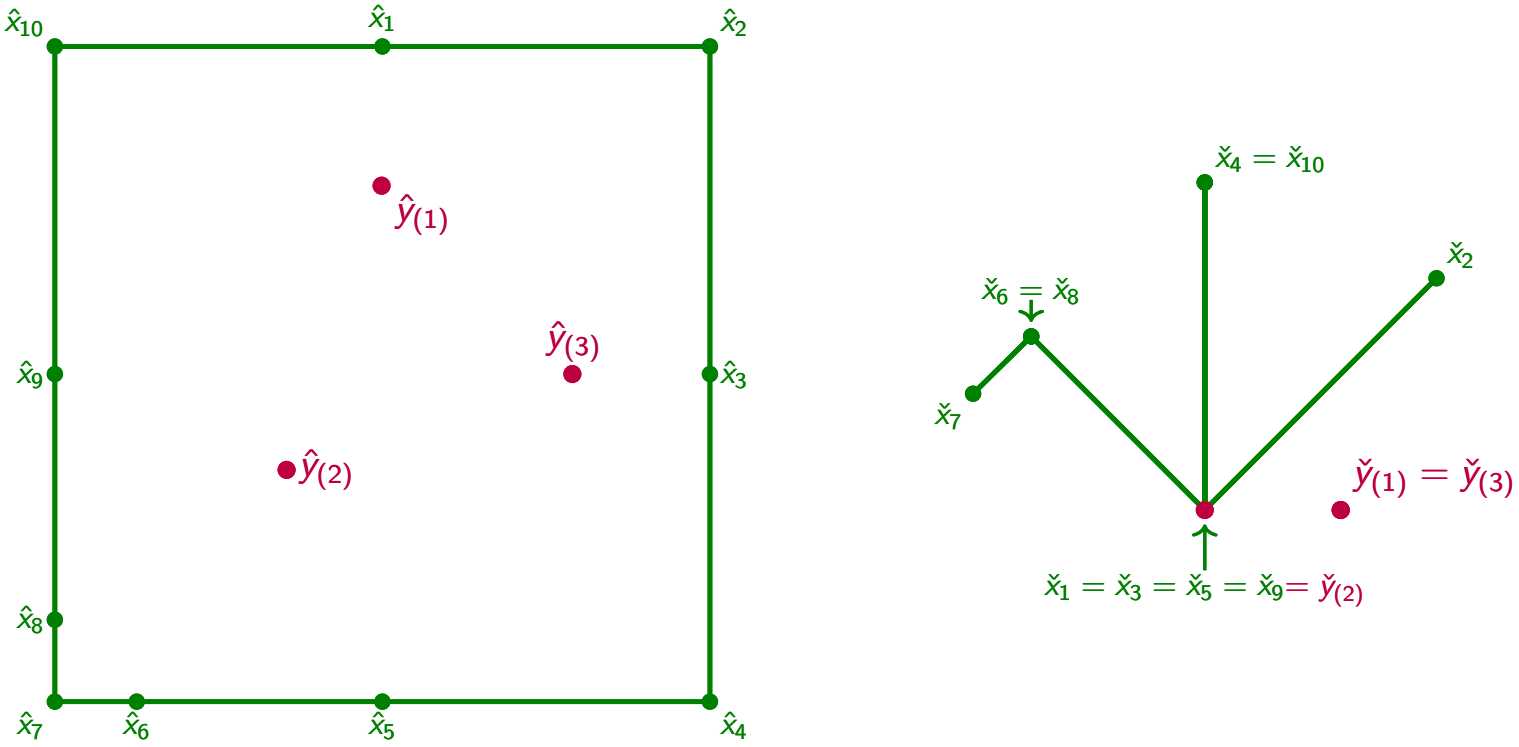

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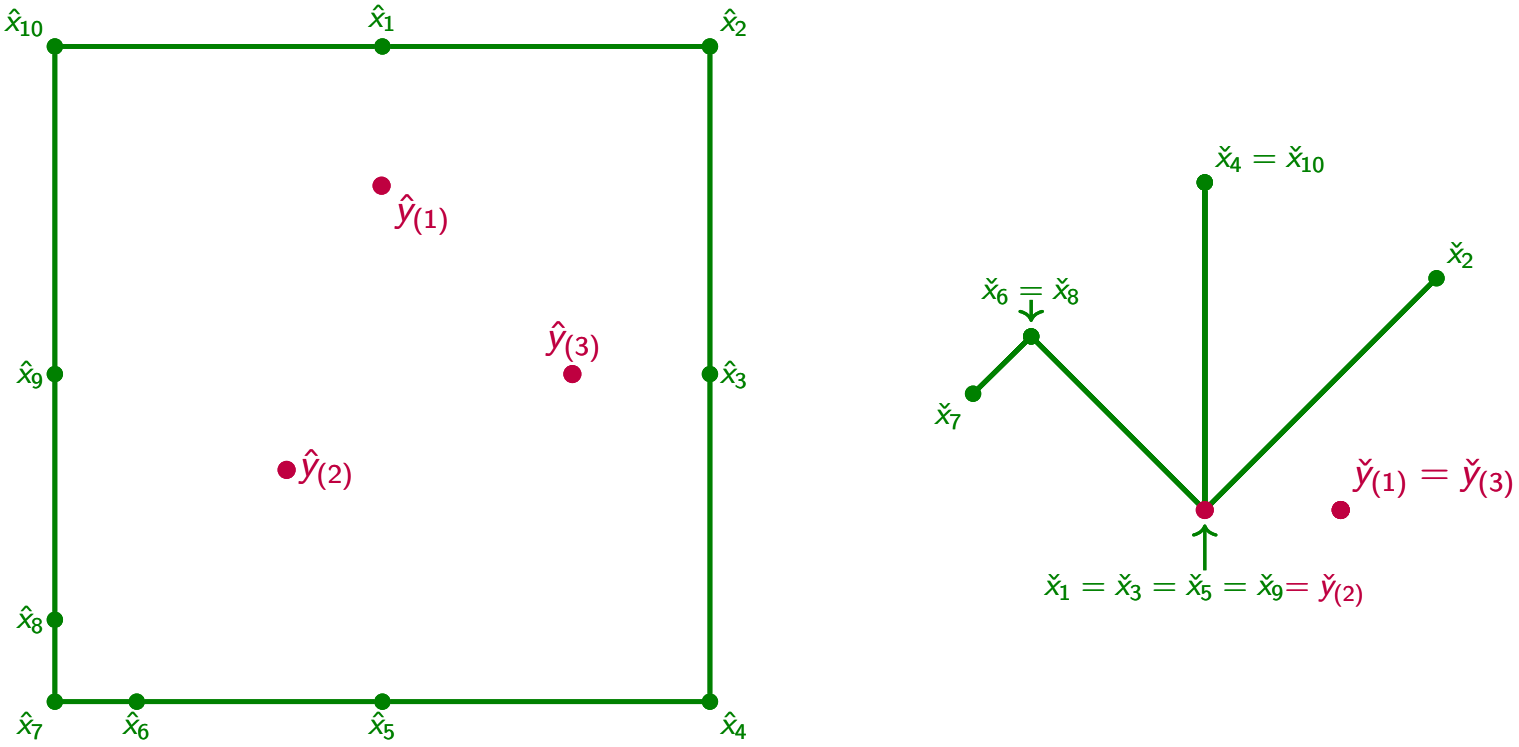

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   (and $|\hat{x}_i(t) - \hat{v}| \geqslant |\check{x}_i(t) - \check{v}|$ for all other vertices $\hat{v}$)

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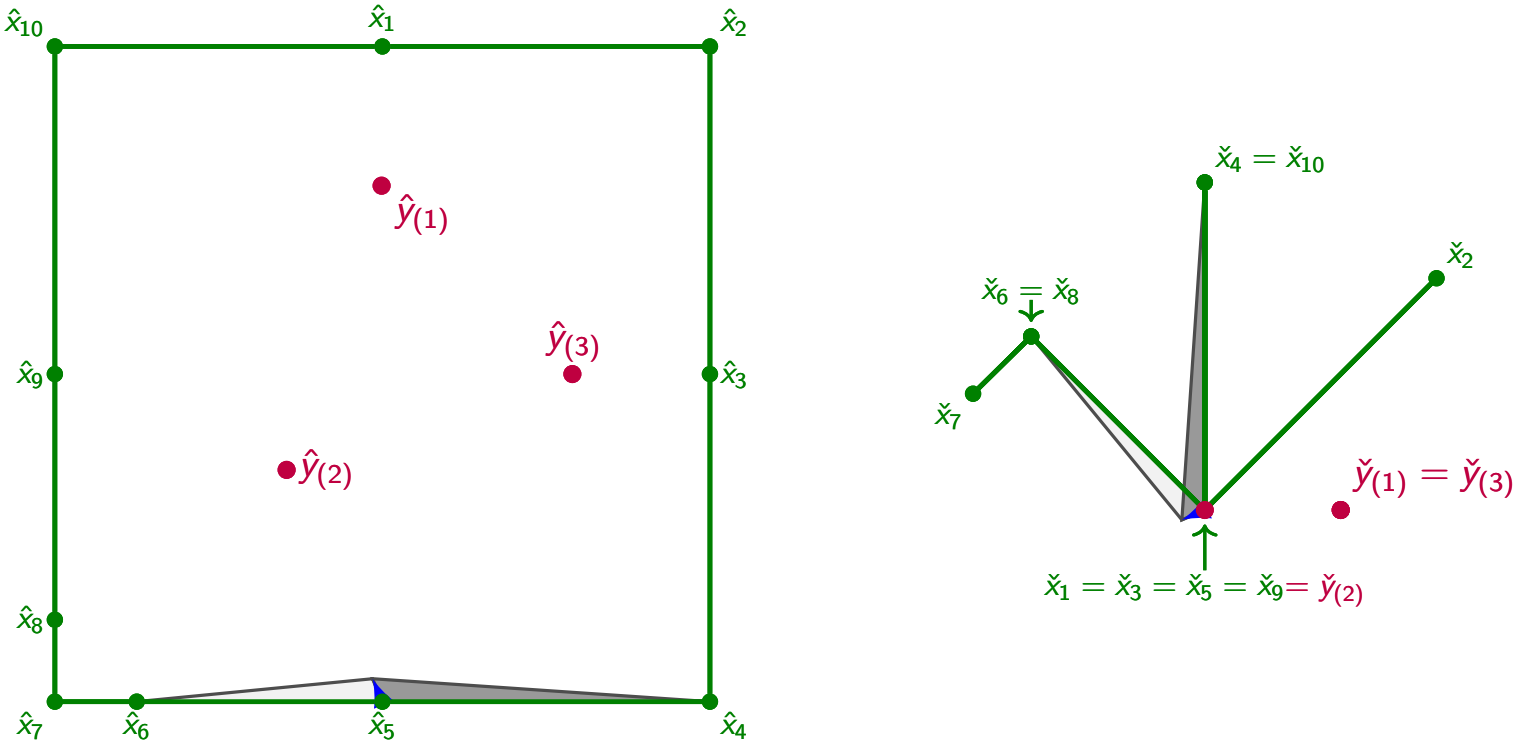

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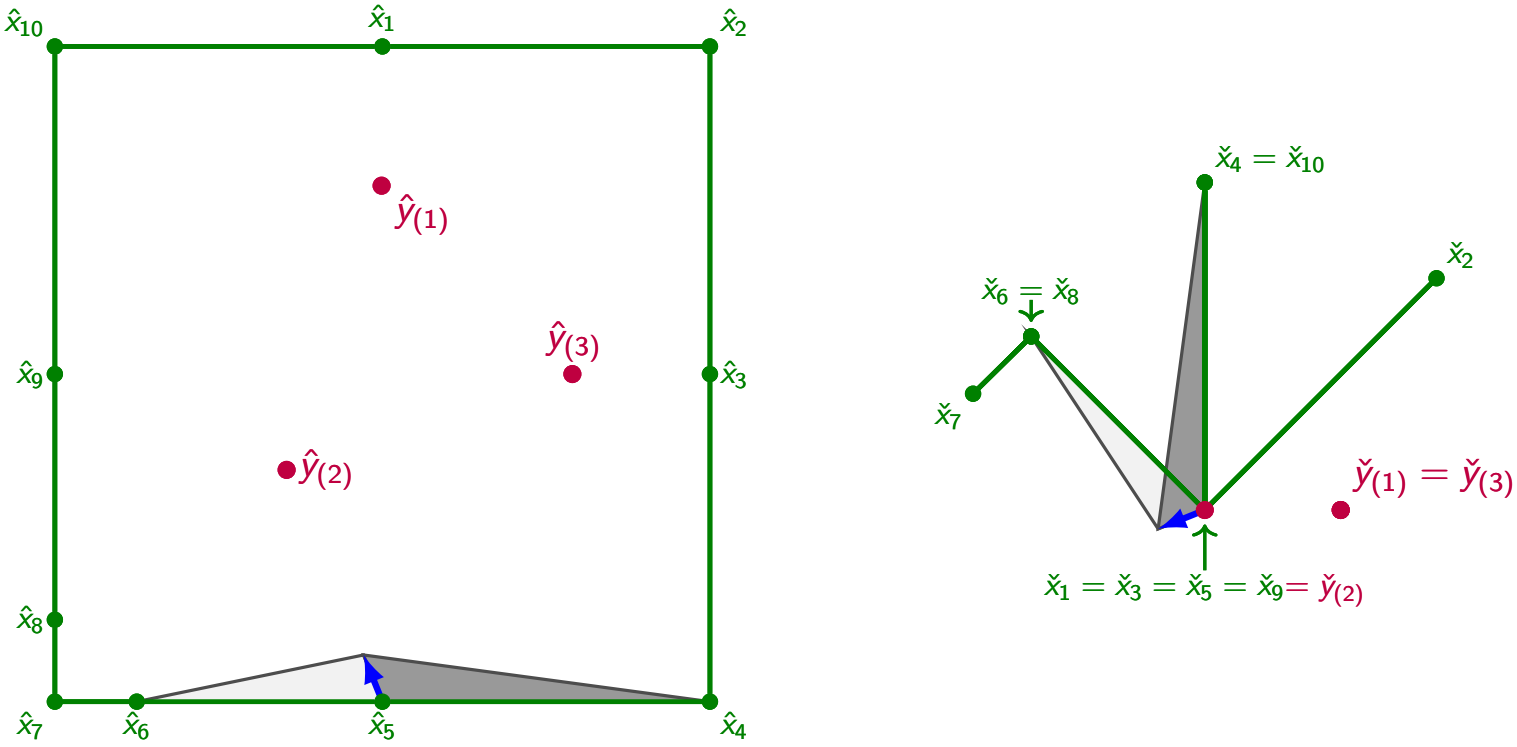

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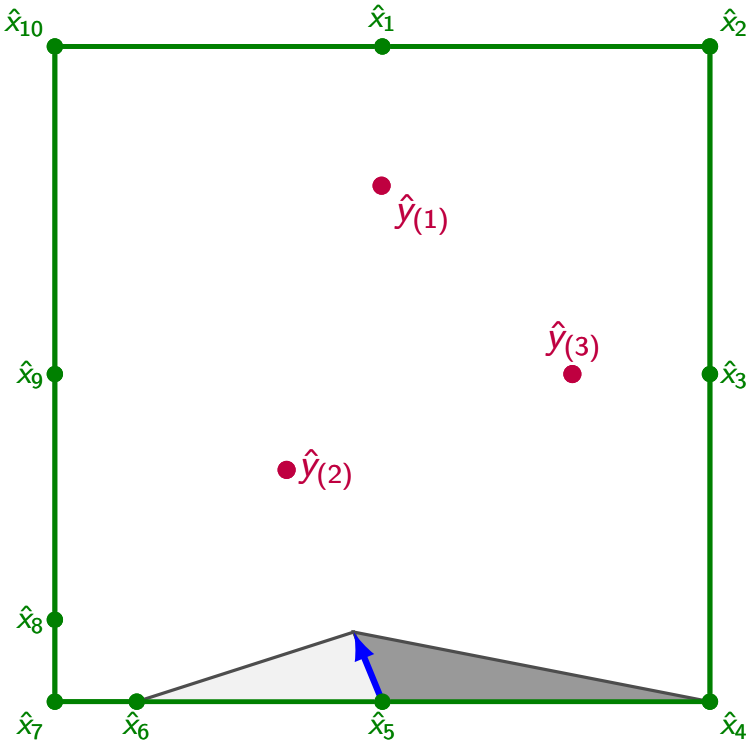

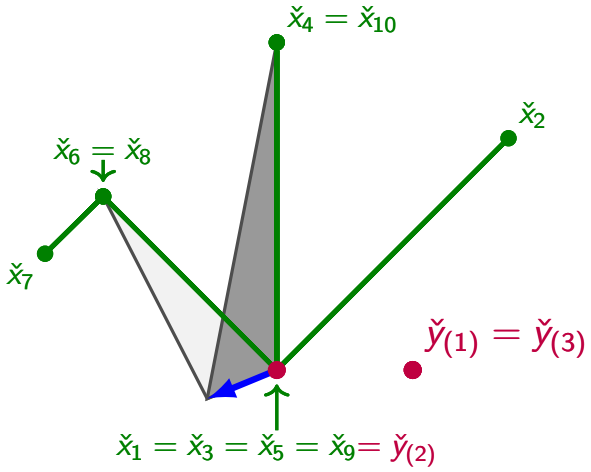

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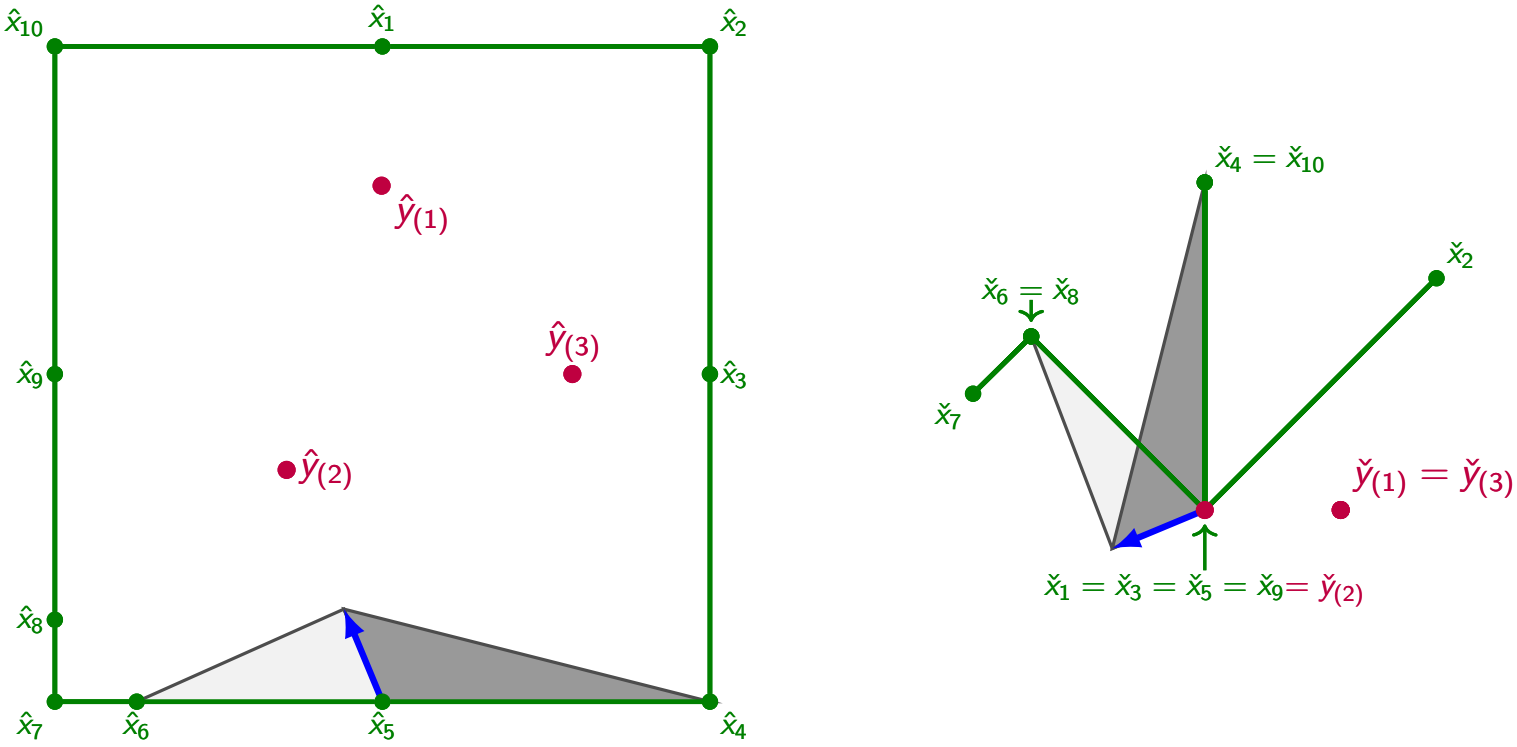

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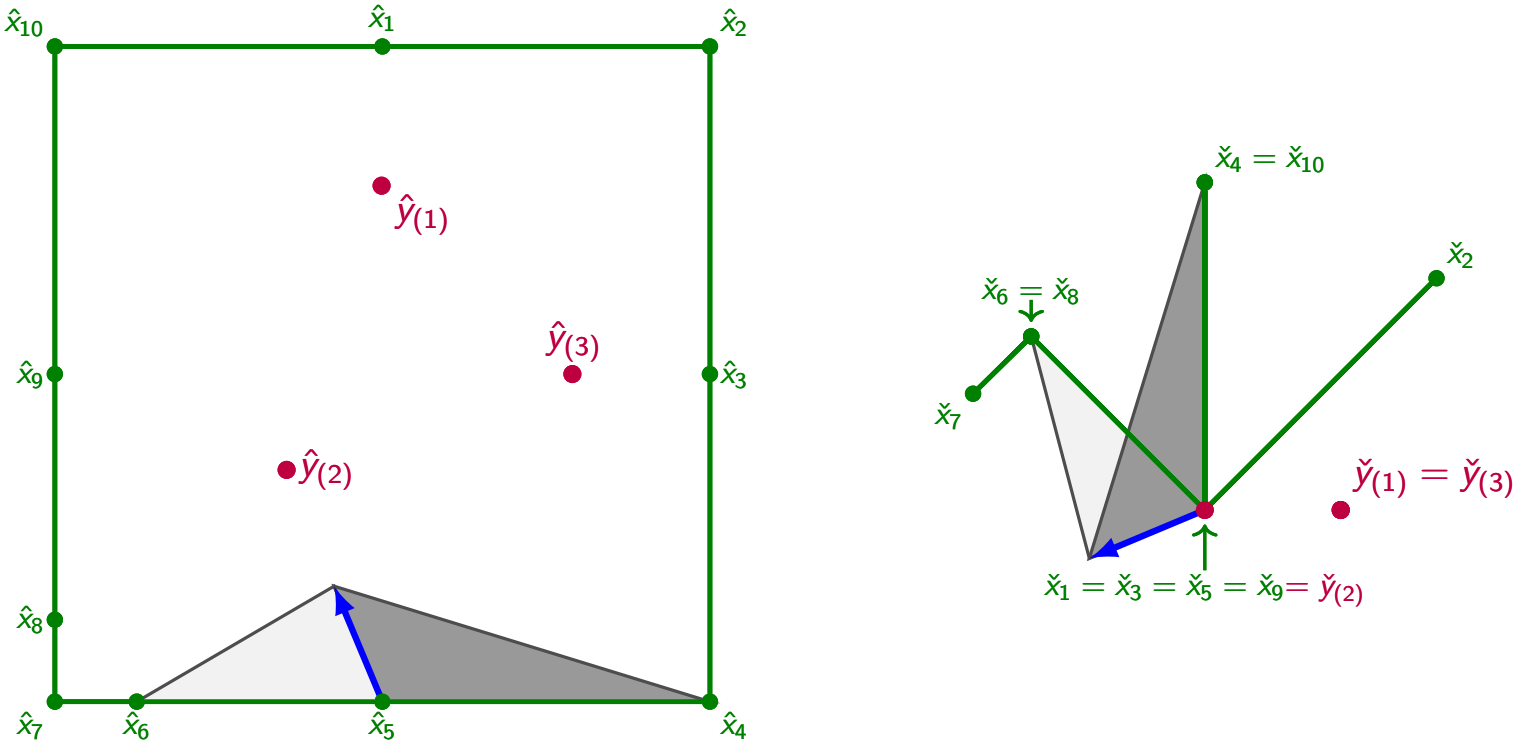

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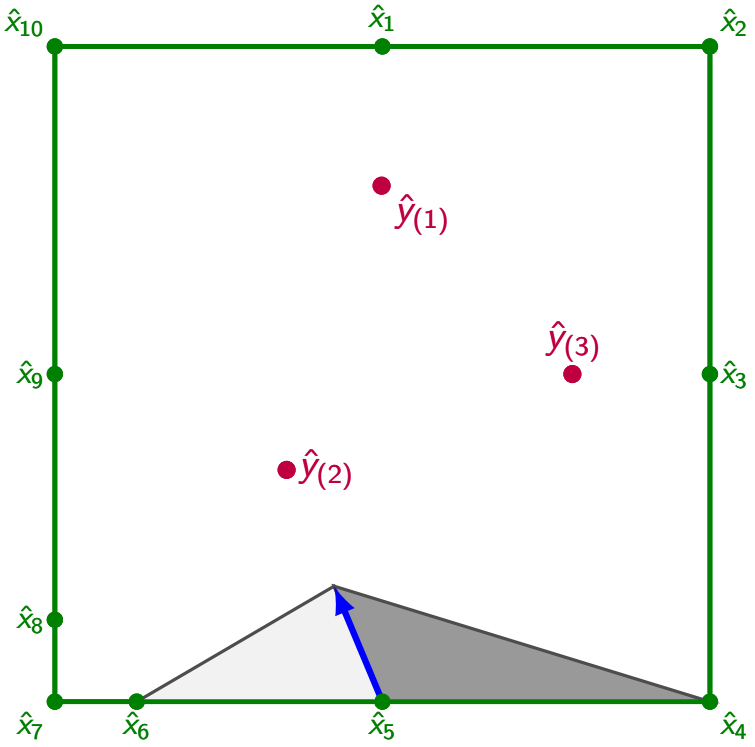

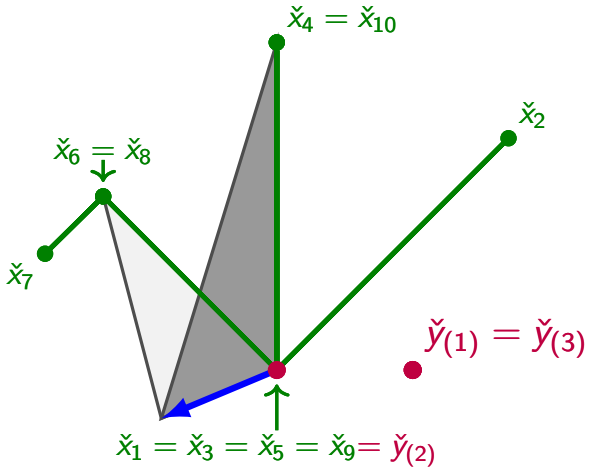

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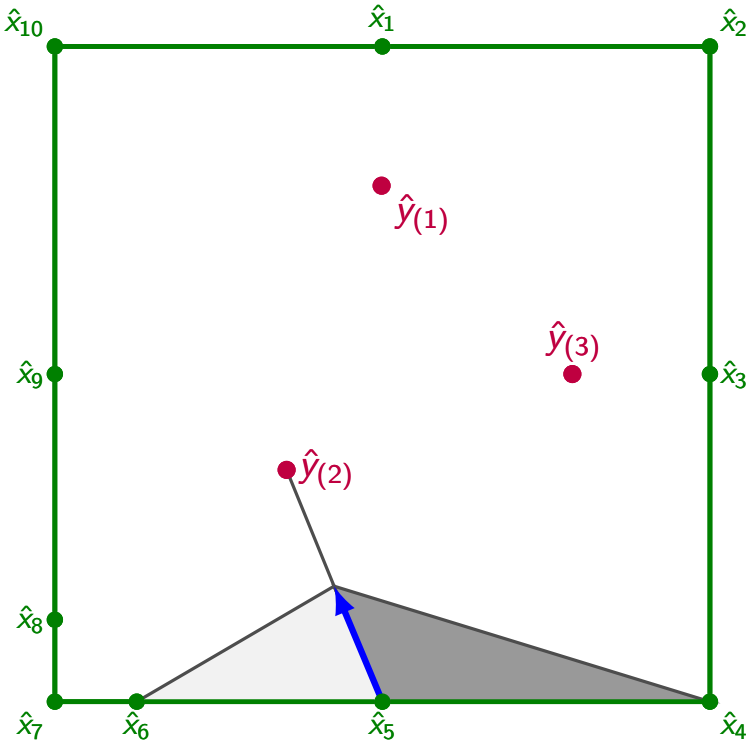

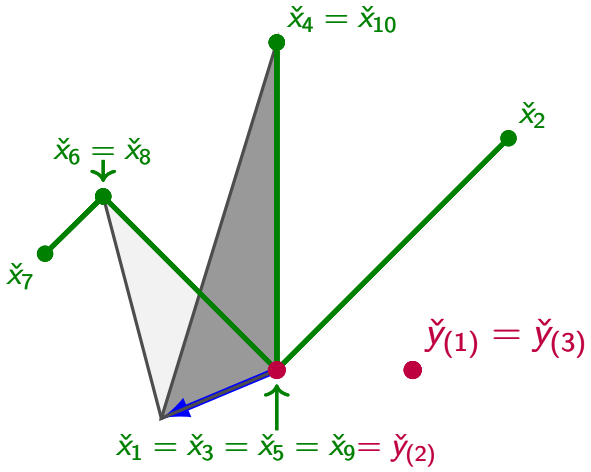

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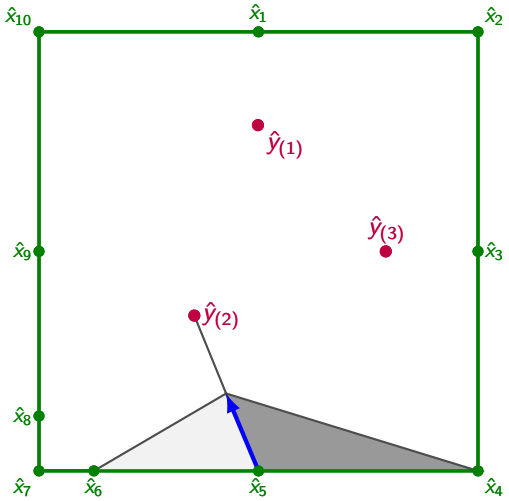

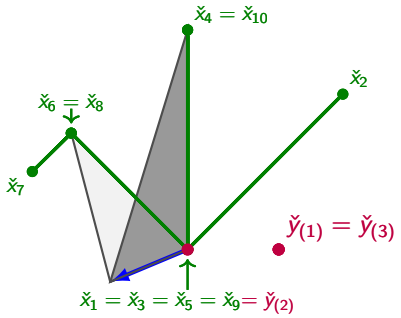

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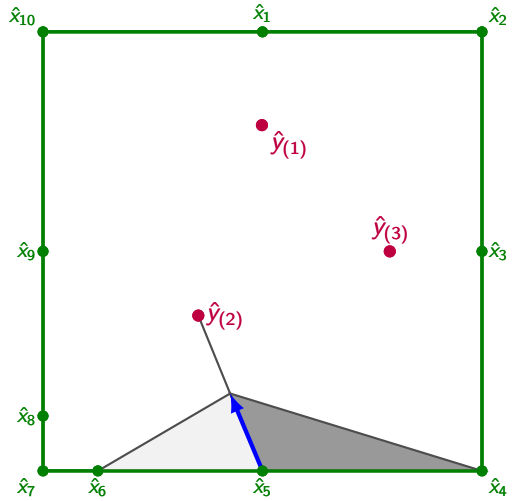

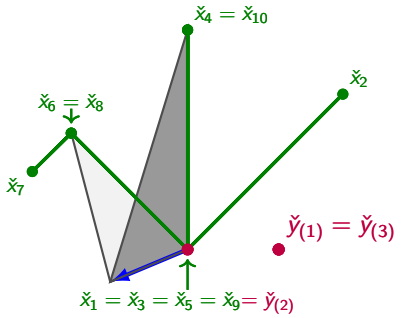

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The Origami Reconstruction Algorithm works.

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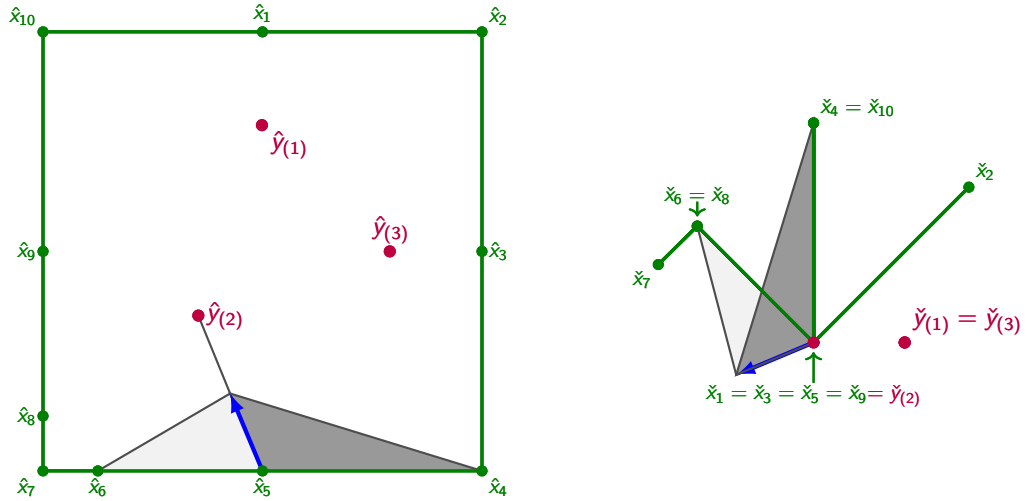

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For all M-positive $(x_1, \ldots, x_n)$, $(y_{(1)}, \ldots, y_{(L)})$, it outputs a valid, embedded OCP.

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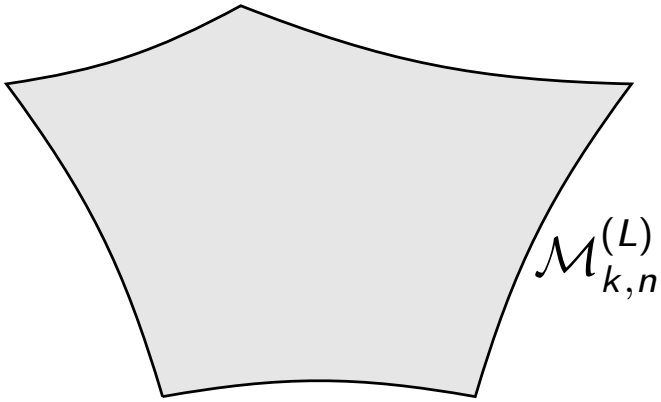

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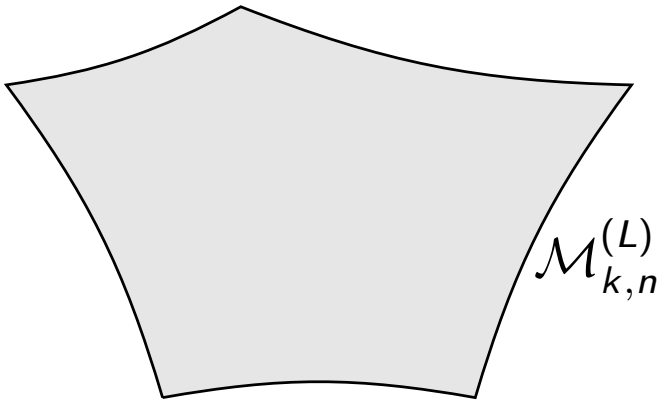

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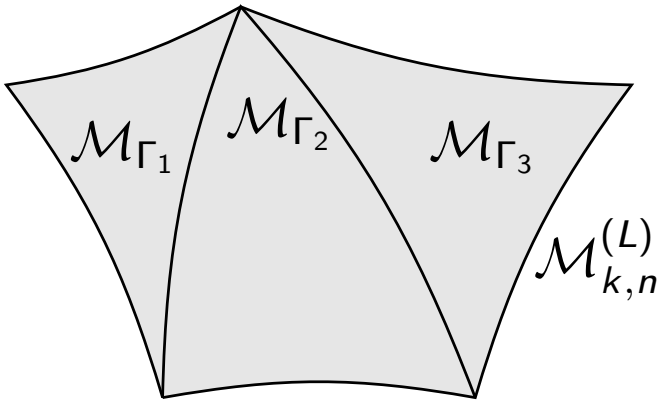

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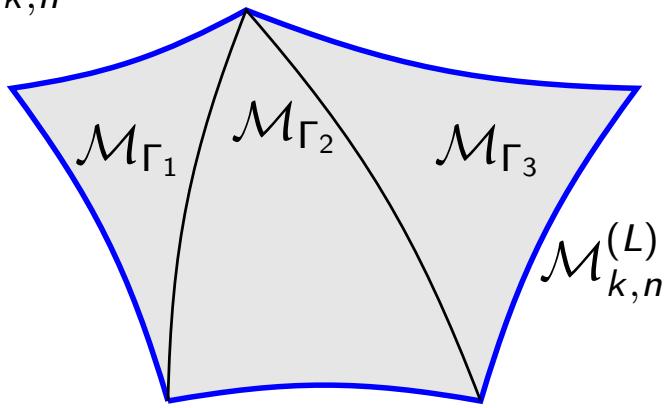

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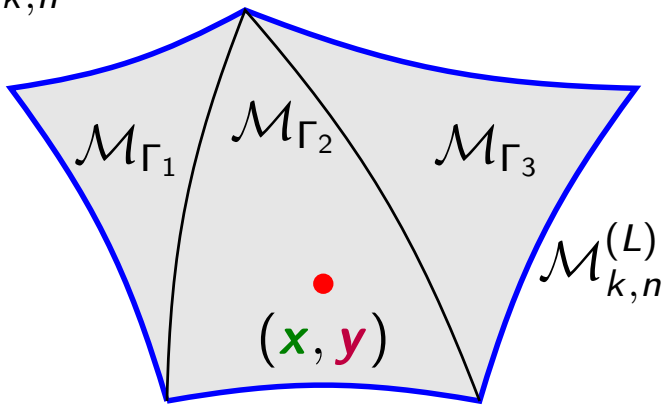

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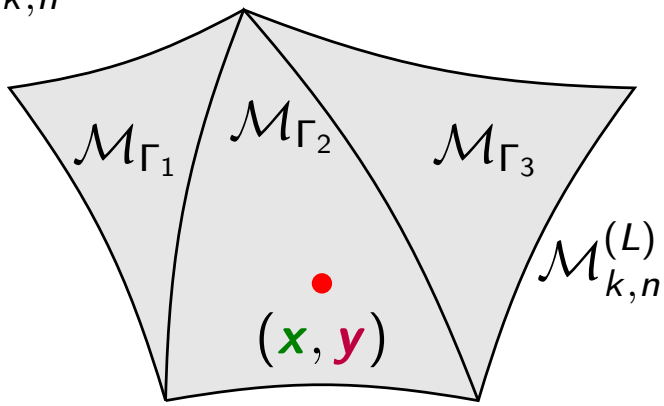

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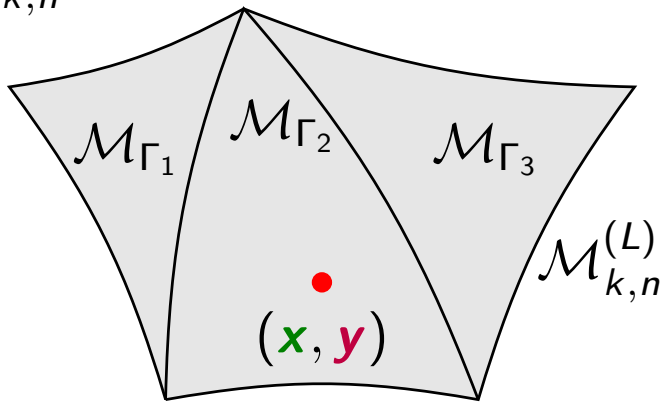

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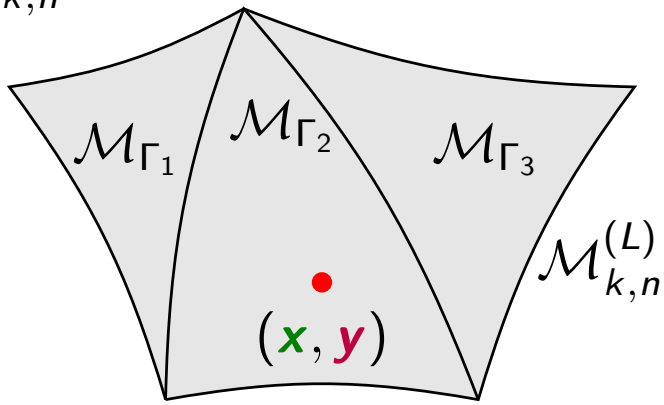

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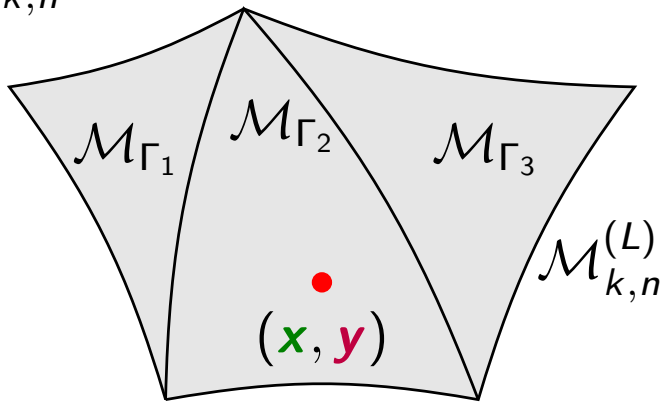

Conjecture (BCFW Triangulation Conjecture [AHT'13])

The BCFW tiles triangulate $\mathcal{M}^{(L)}_{k,n}$ and $\mathcal{A}^{(L)}_{k-2,n}$.

- **Loop BCFW recursion:** $A^{(L)}_{k,n} = \sum_{\substack{\text{planar bicolored graph } \Gamma \in \mathbf{\Gamma}^{\text{BCFW}}_{k,n,(L)} \\ \text{with } L \text{ marked faces}}} A^{(L)}_{\Gamma}.$

  [Britto–Cachazo–Feng–Witten '05]

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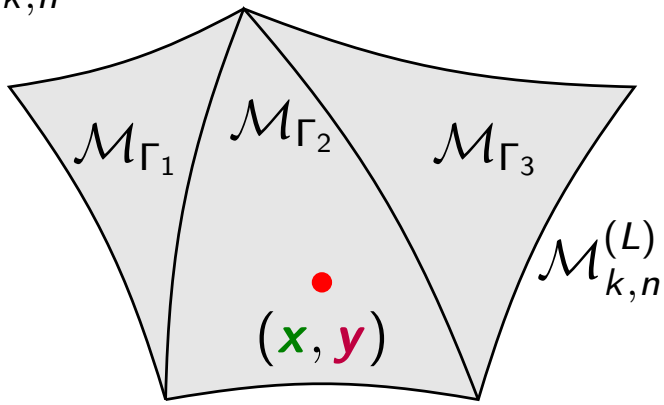

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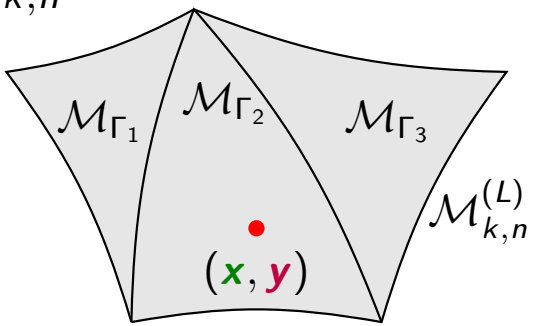

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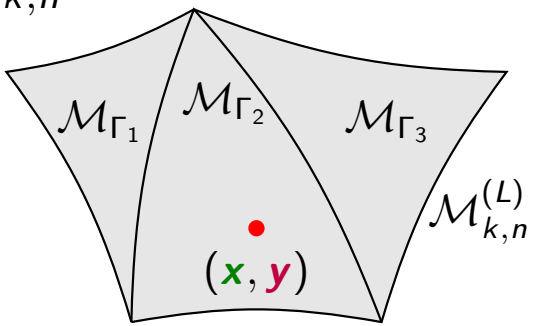

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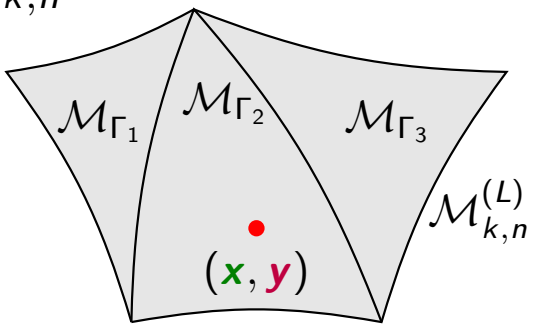

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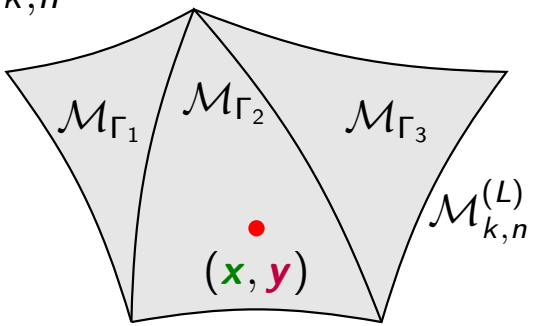

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- BCFW Triangulation Conjecture $\iff$ Origami Reconstruction Algorithm works.

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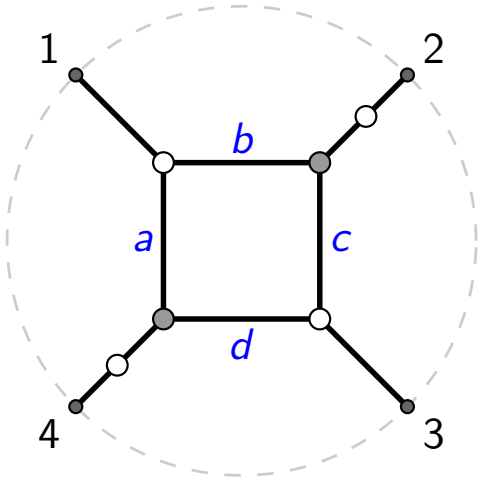

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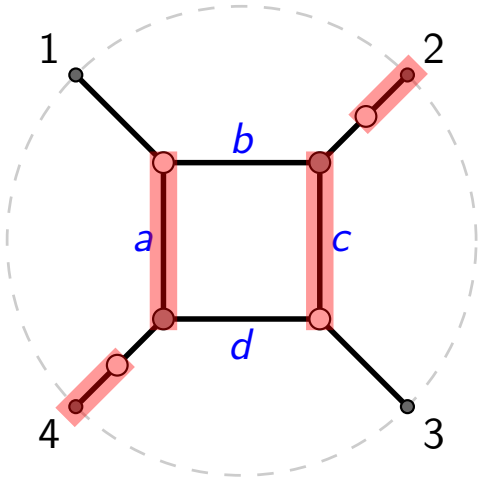

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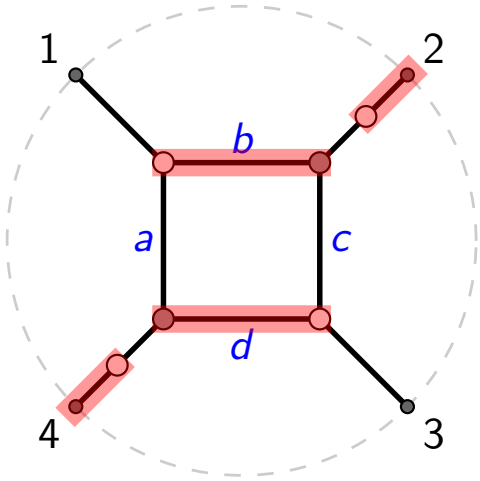

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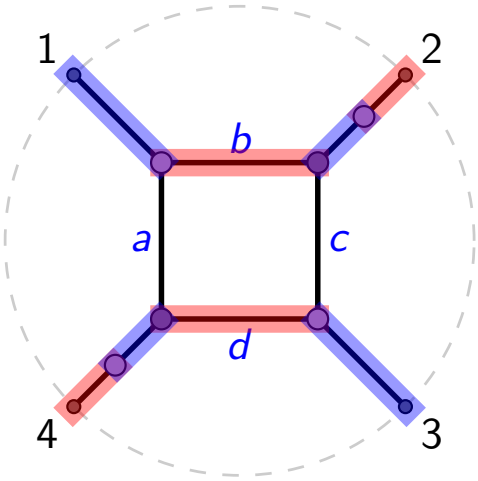

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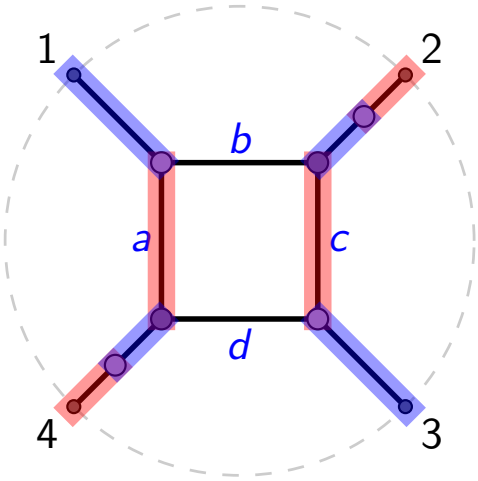

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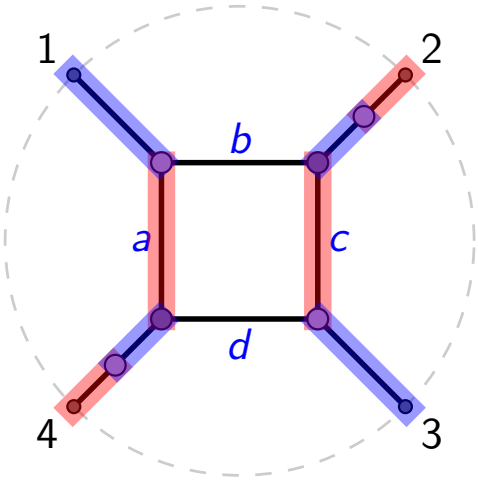

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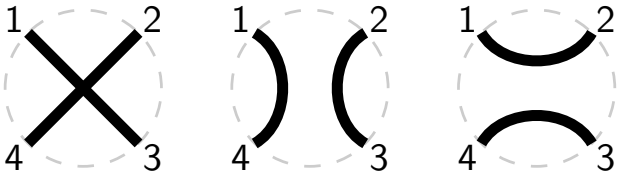

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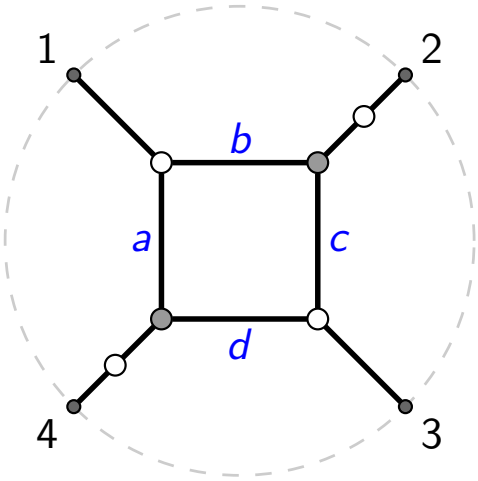

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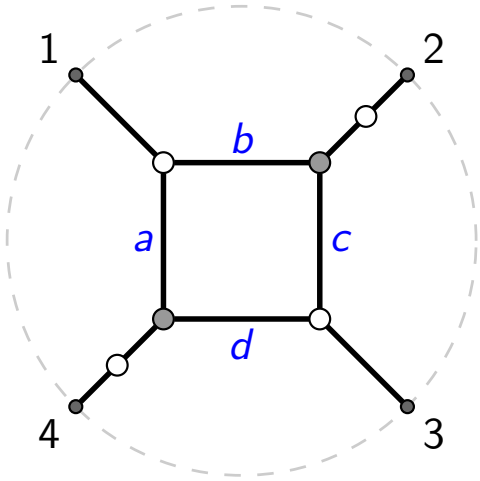

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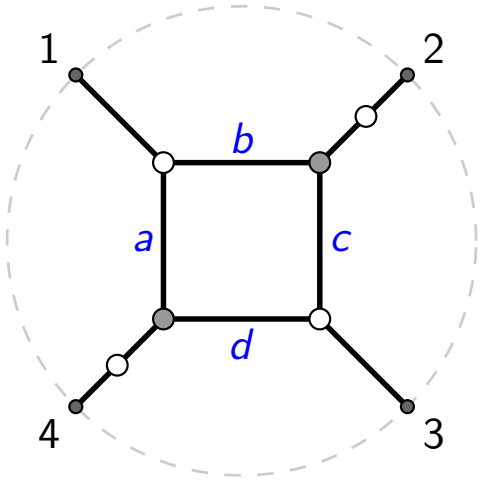

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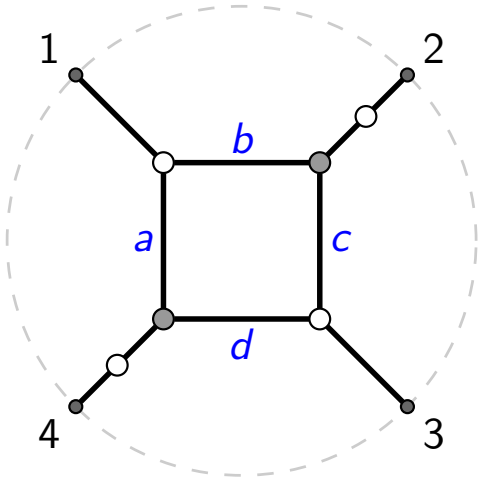

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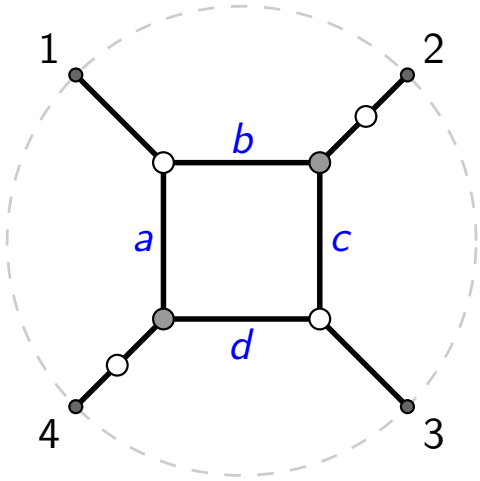

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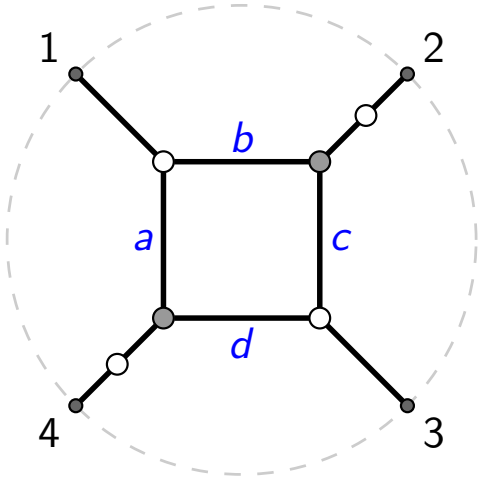

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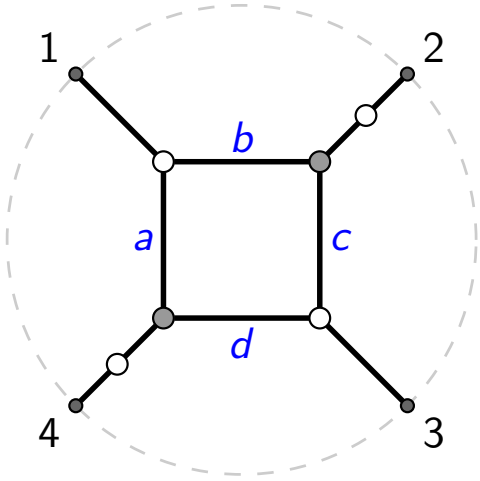

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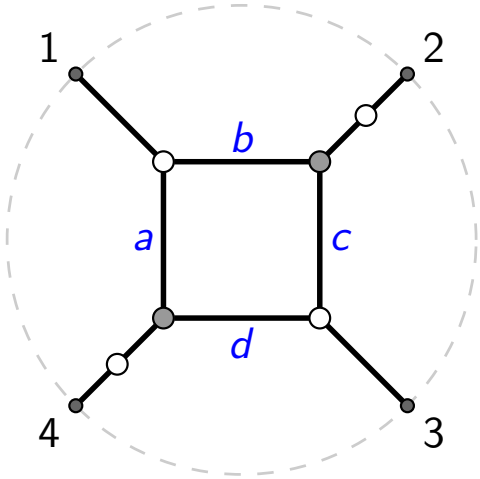

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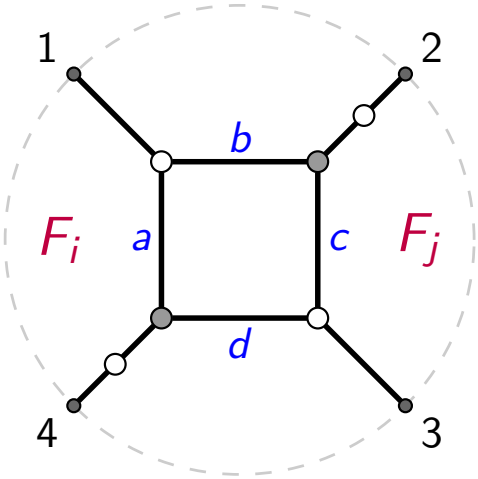

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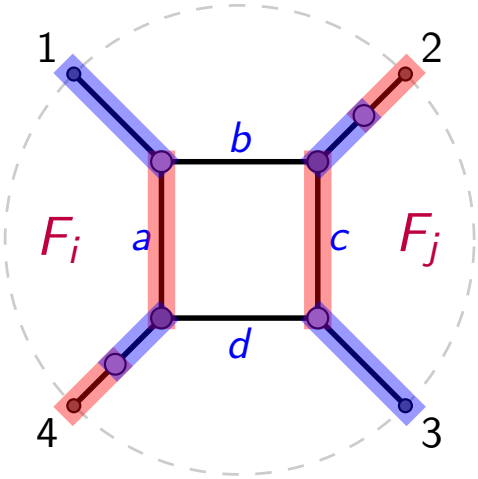

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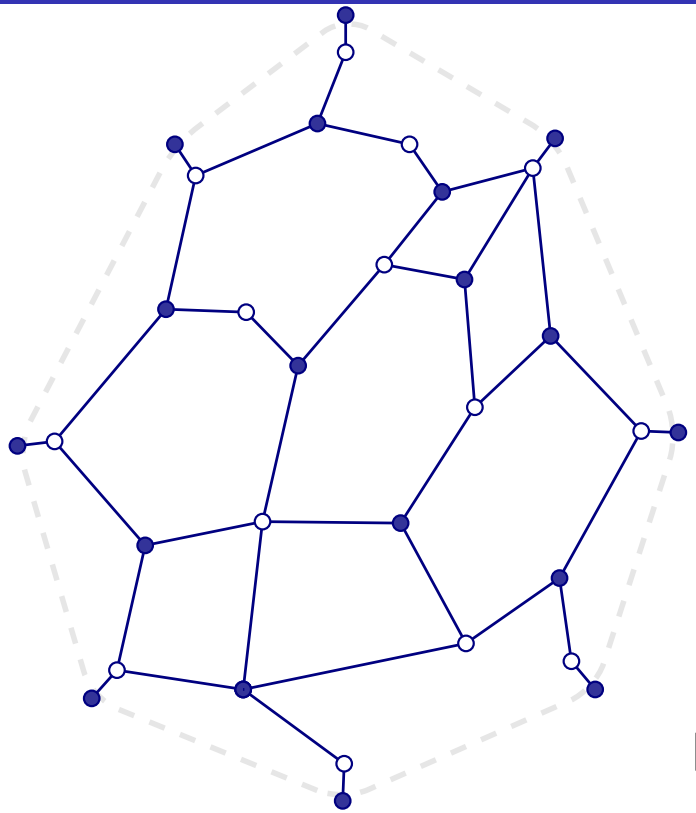
T-duality for planar bipartite graphs



Γ

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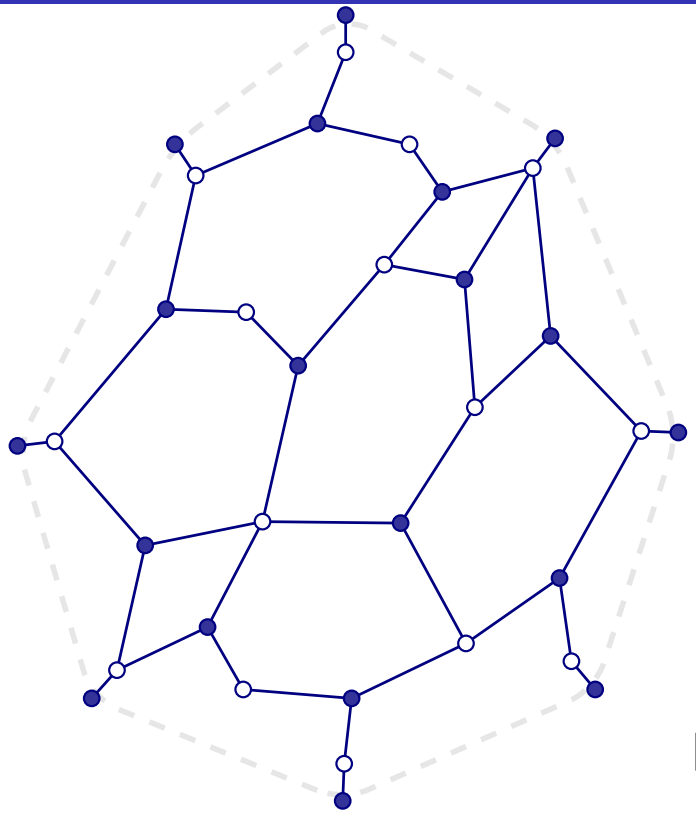

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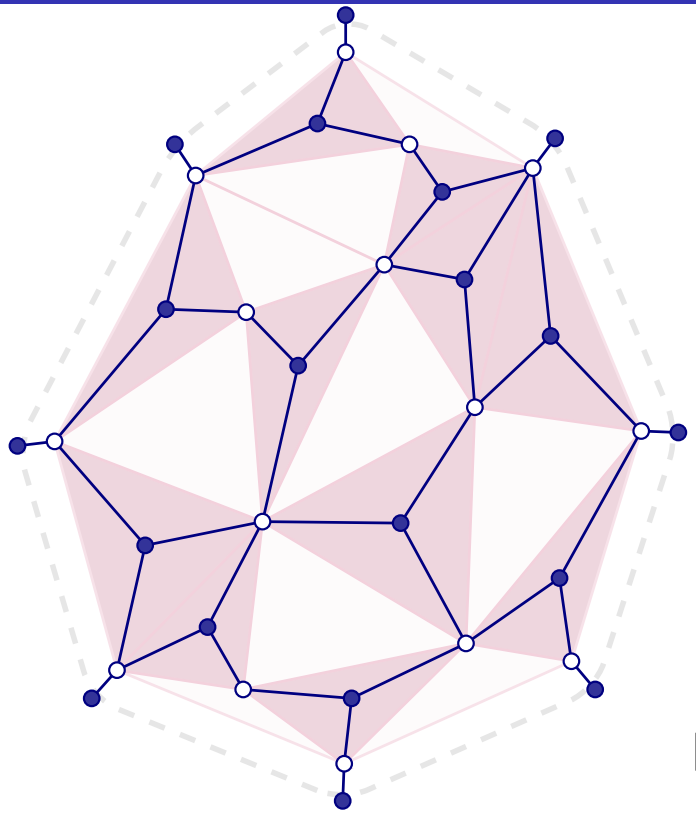

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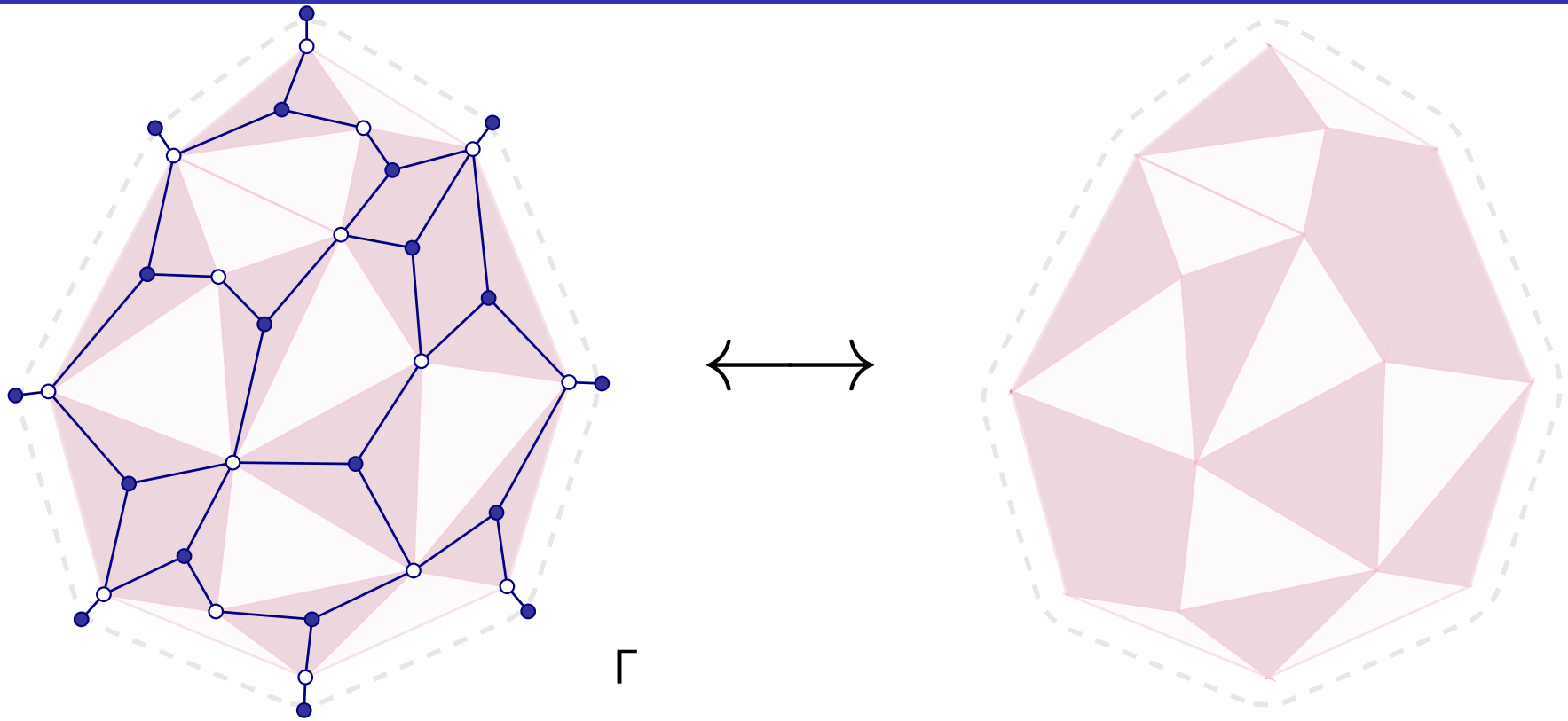

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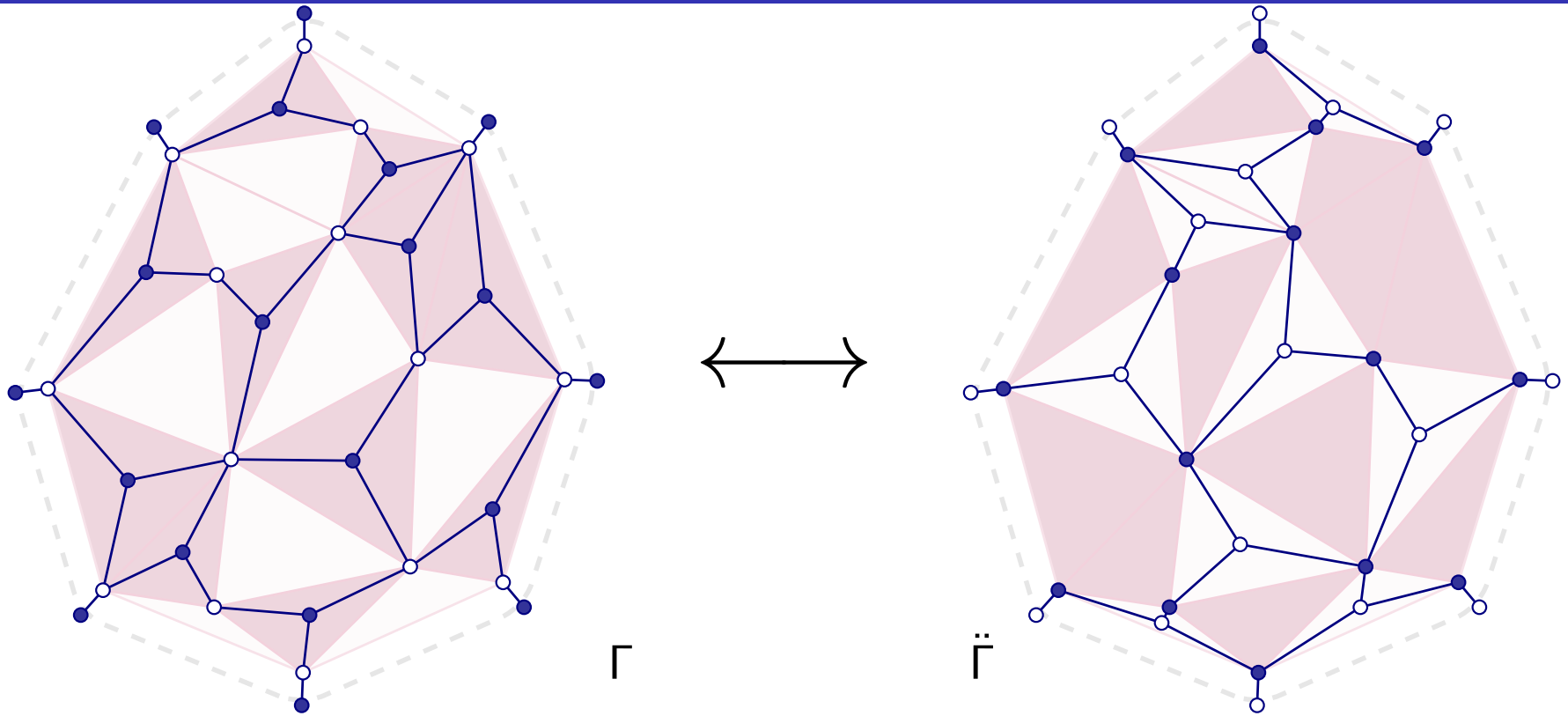

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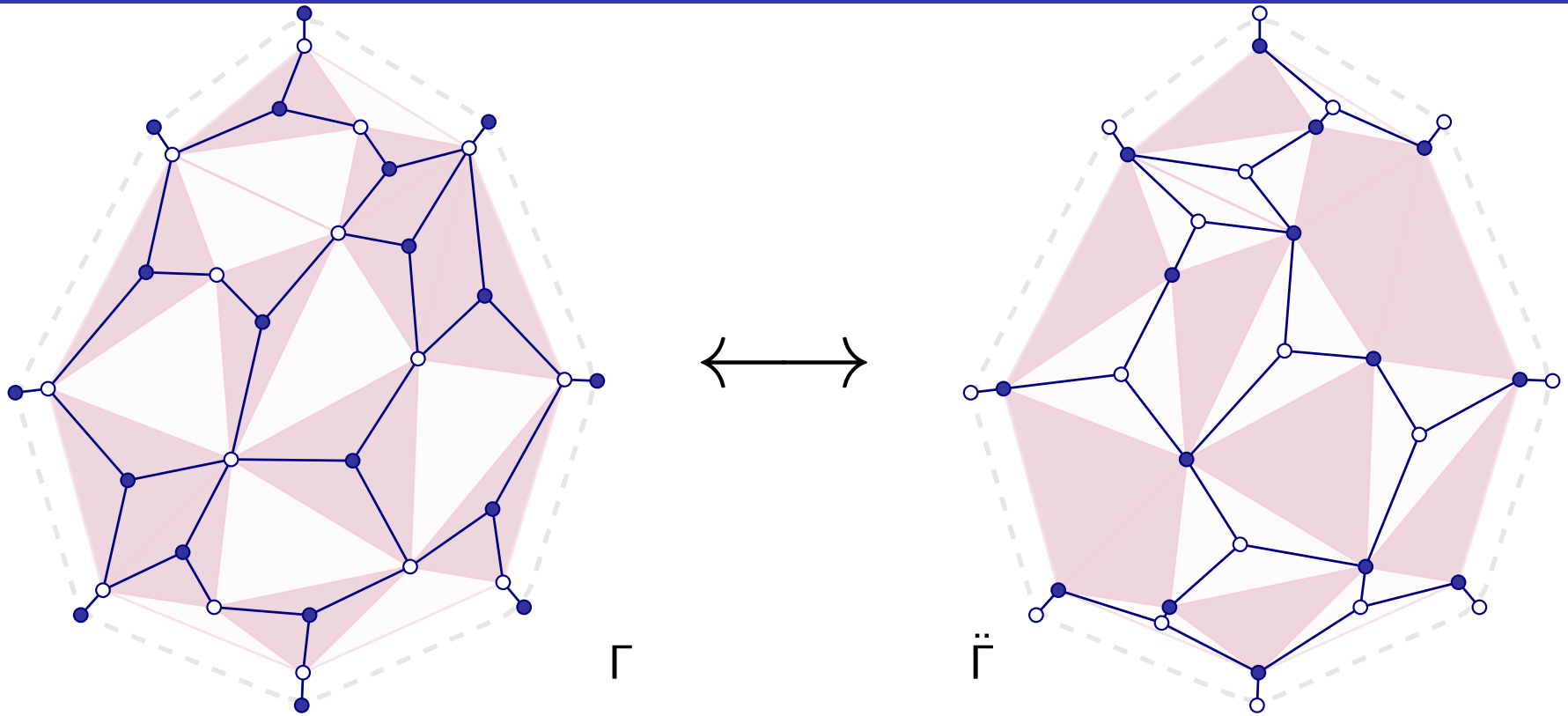

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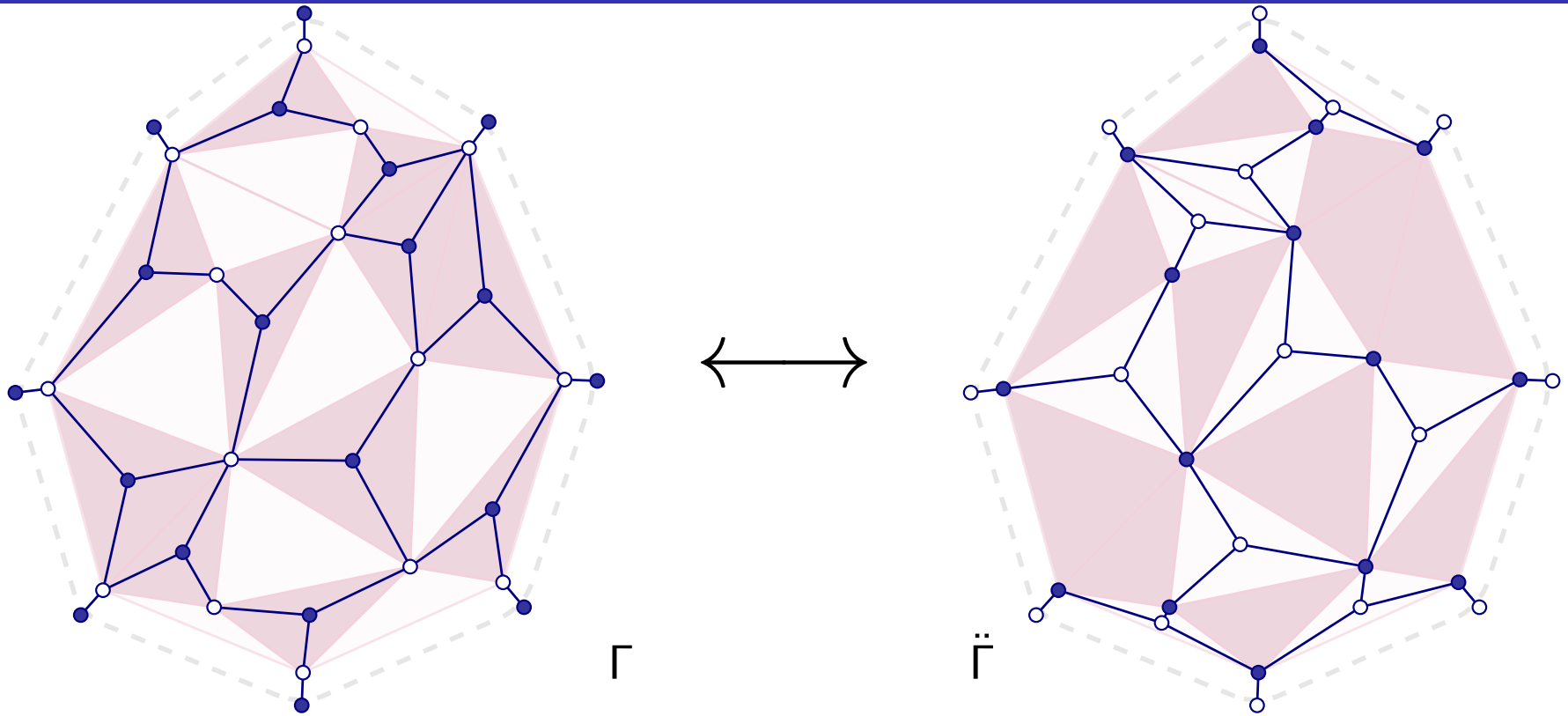

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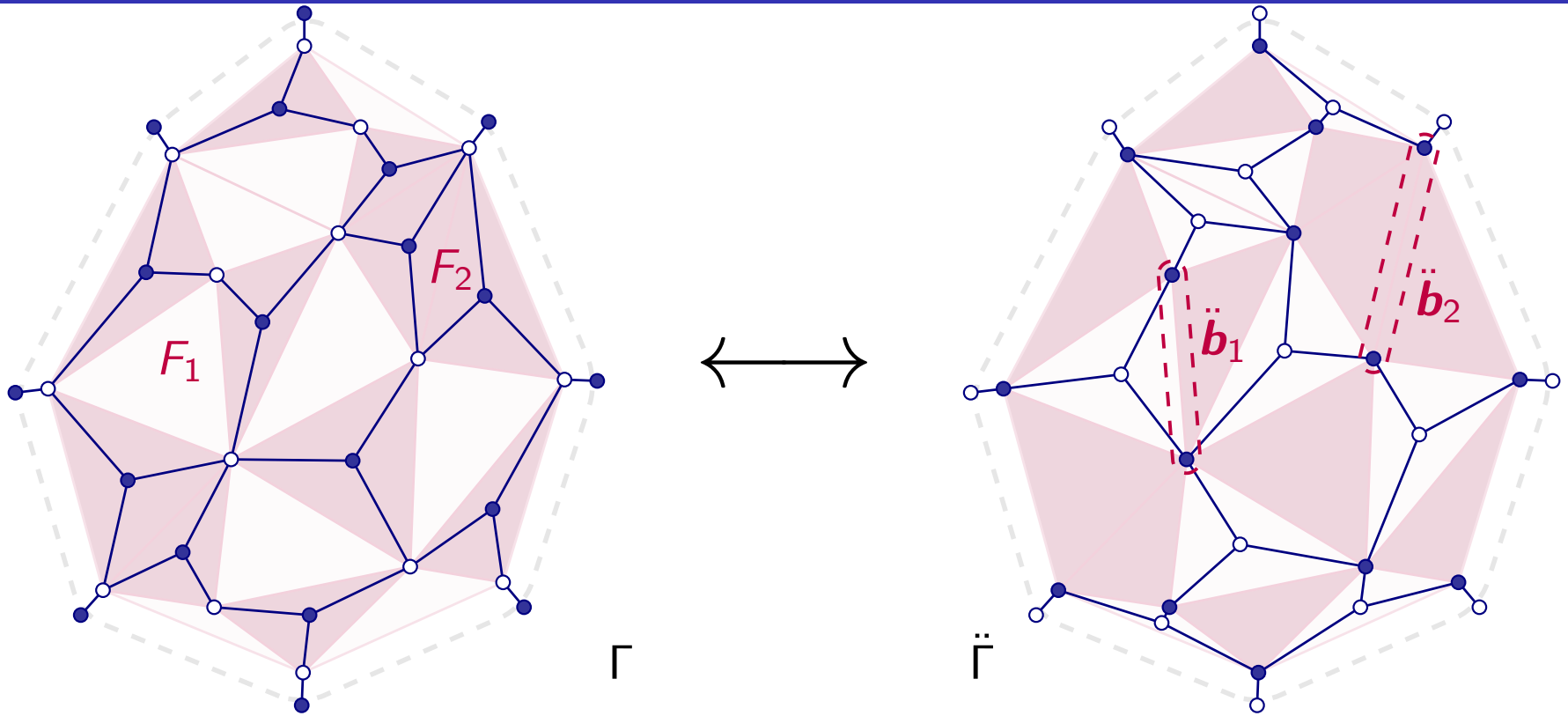

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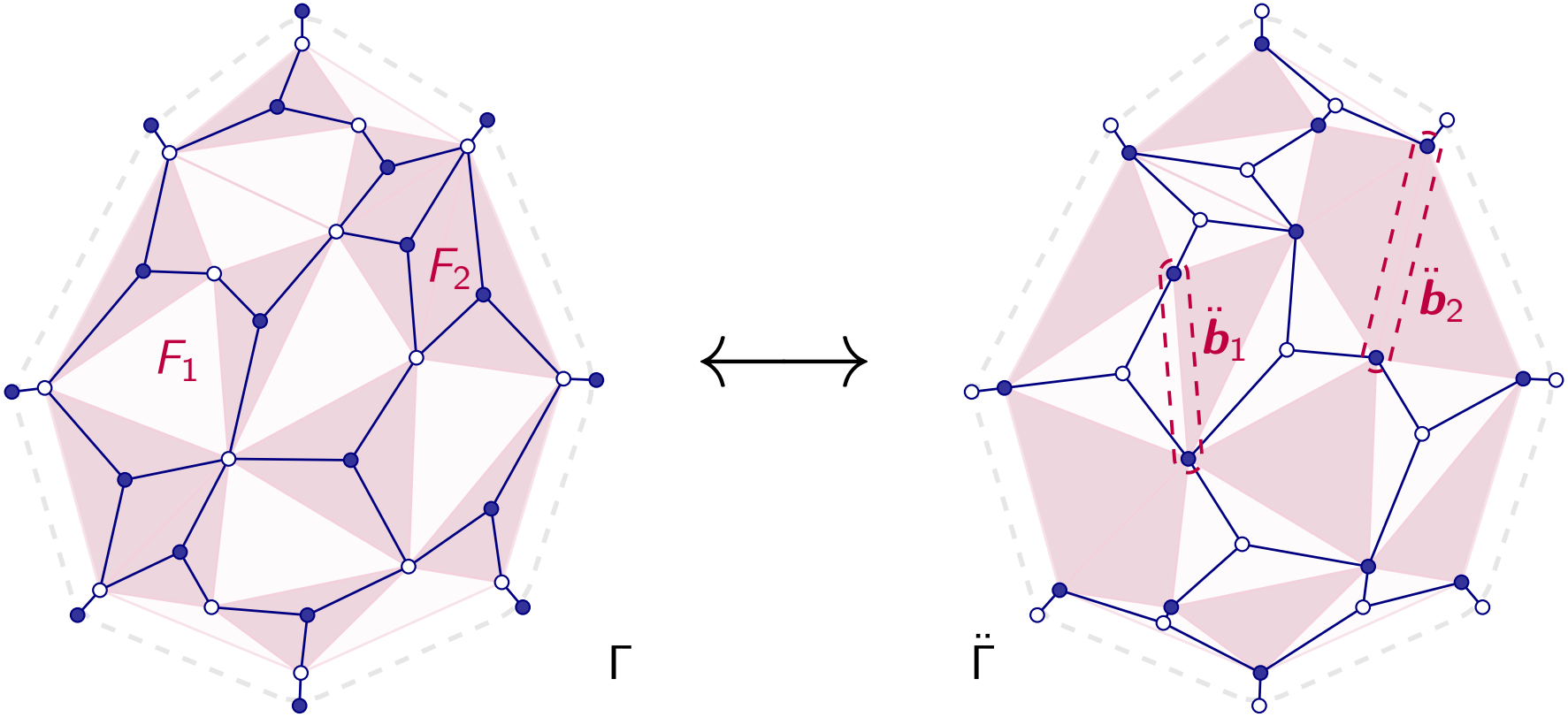
F_1
F_2
Γ
$\ddot{b}_1$
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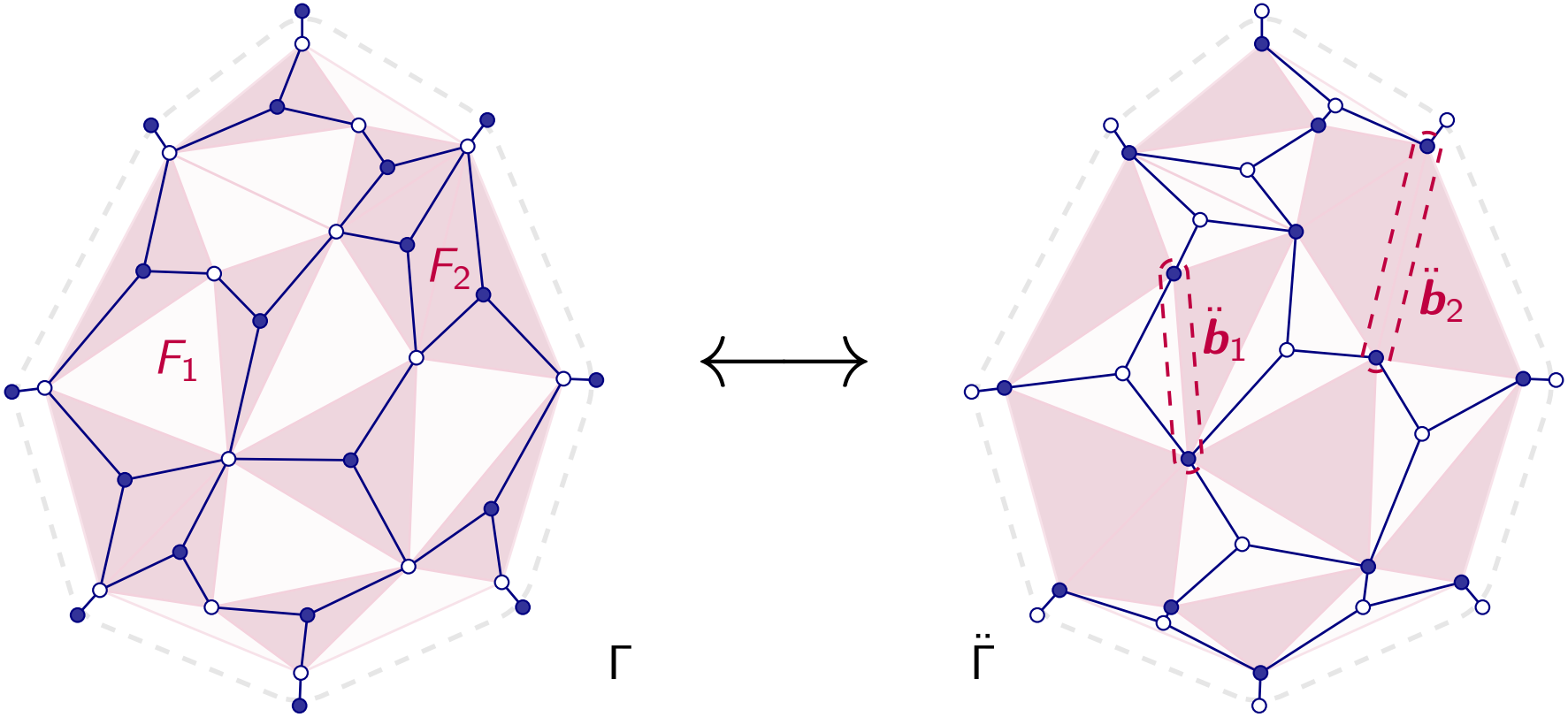
$\ddot{\Gamma}$



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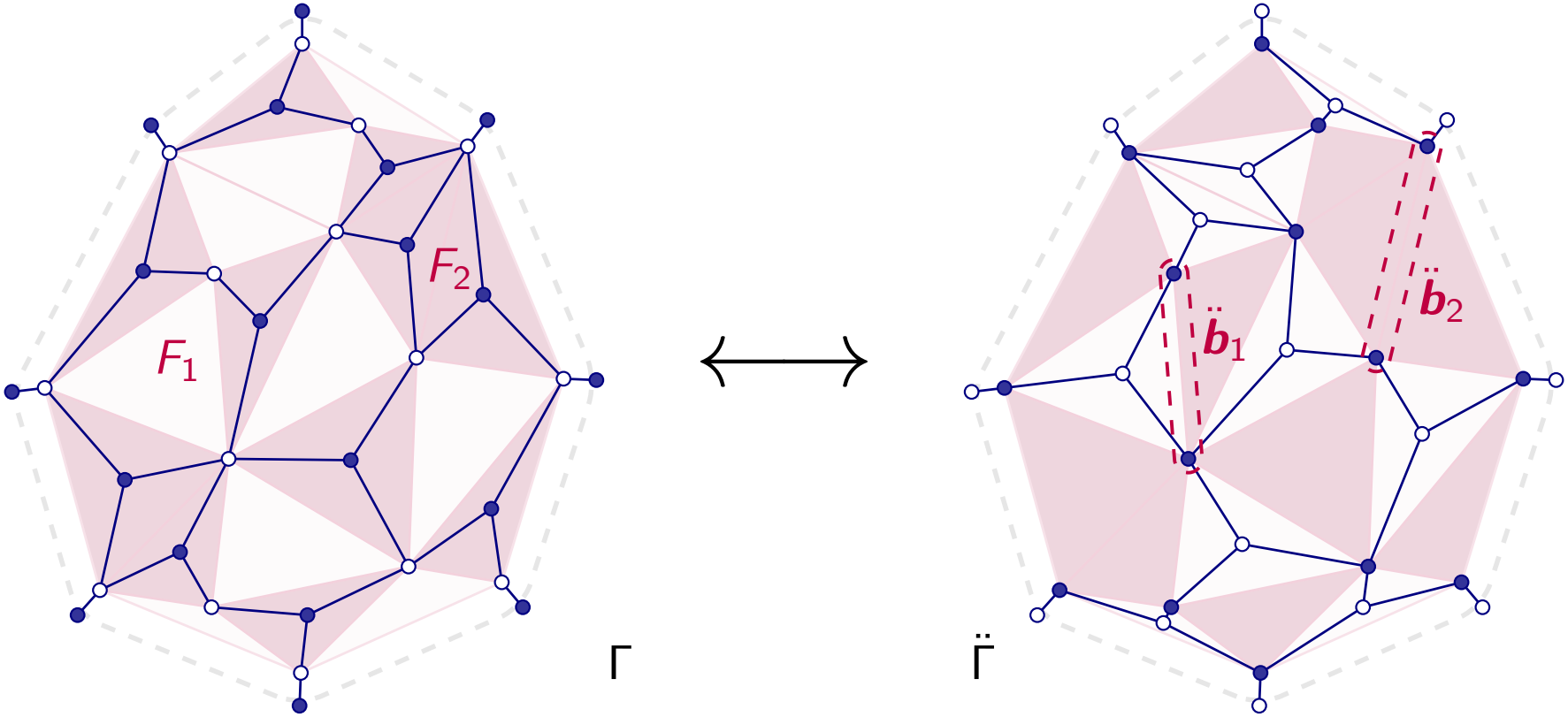

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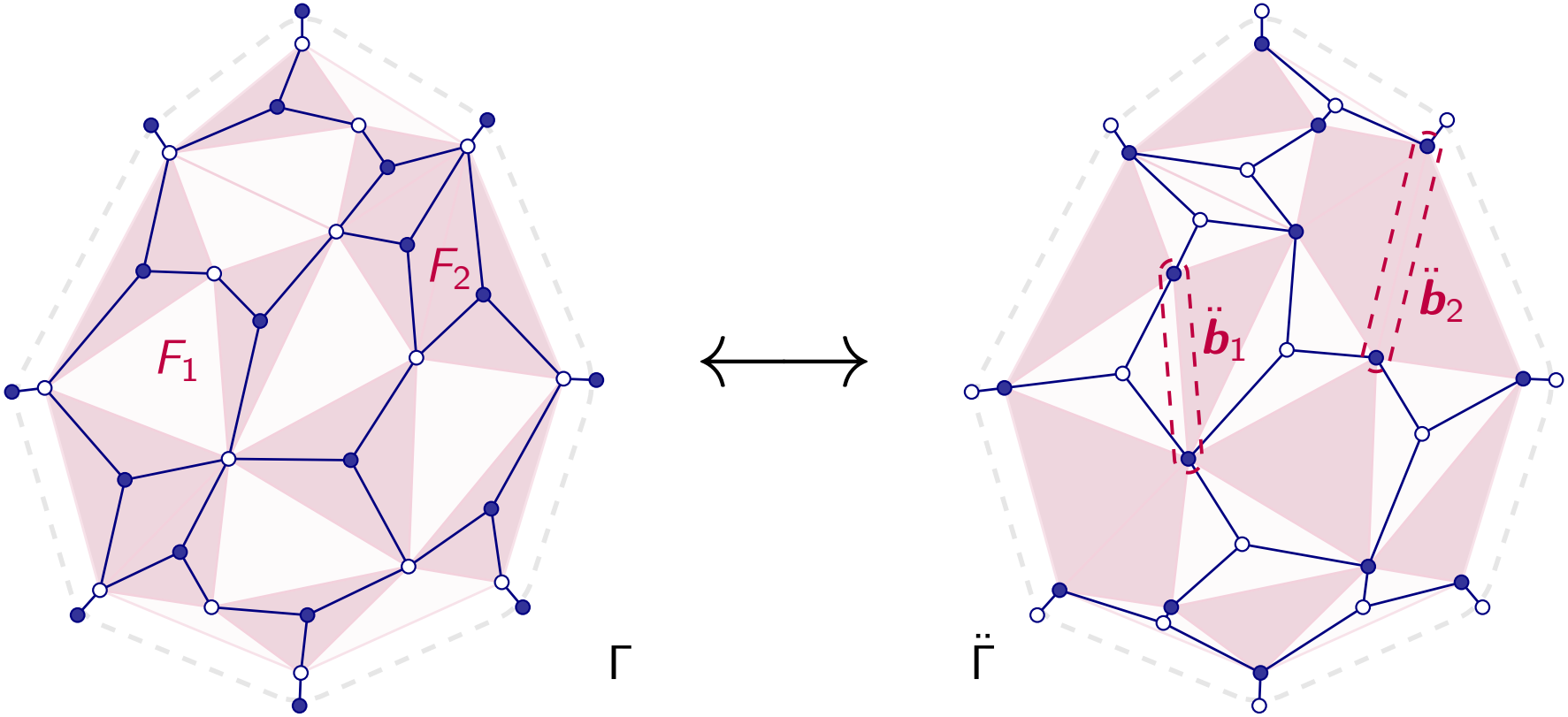
- *Boundary measurements of (Γ, wt) and $(\ddot{\Gamma}, \ddot{\mathrm{wt}})$ are related by the magic projector $\mathrm{Gr}_{\geqslant 0}(k, n) \to \mathrm{Gr}_{\geqslant 0}(k-2, n)$ of [Arkani-Hamed–Cachazo–Cheung '10].*



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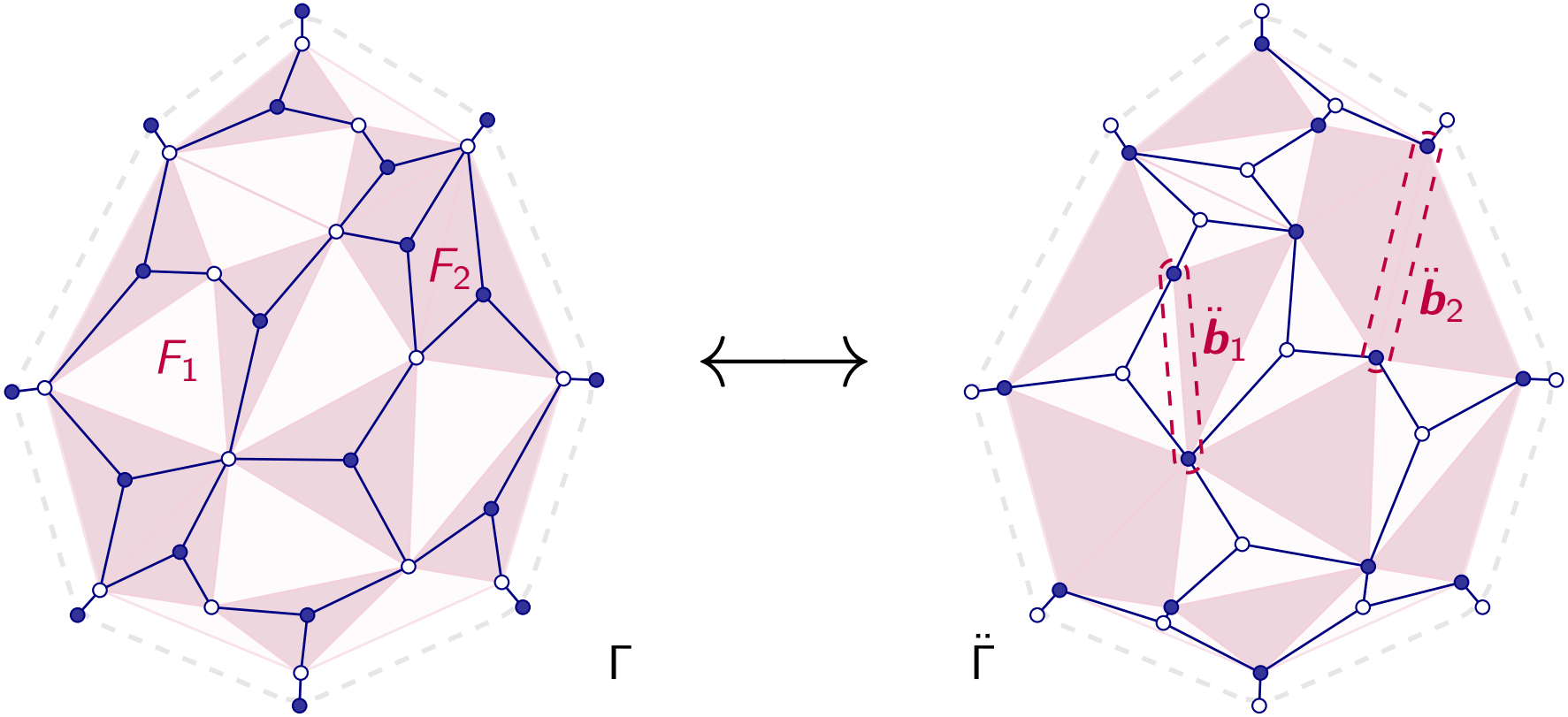

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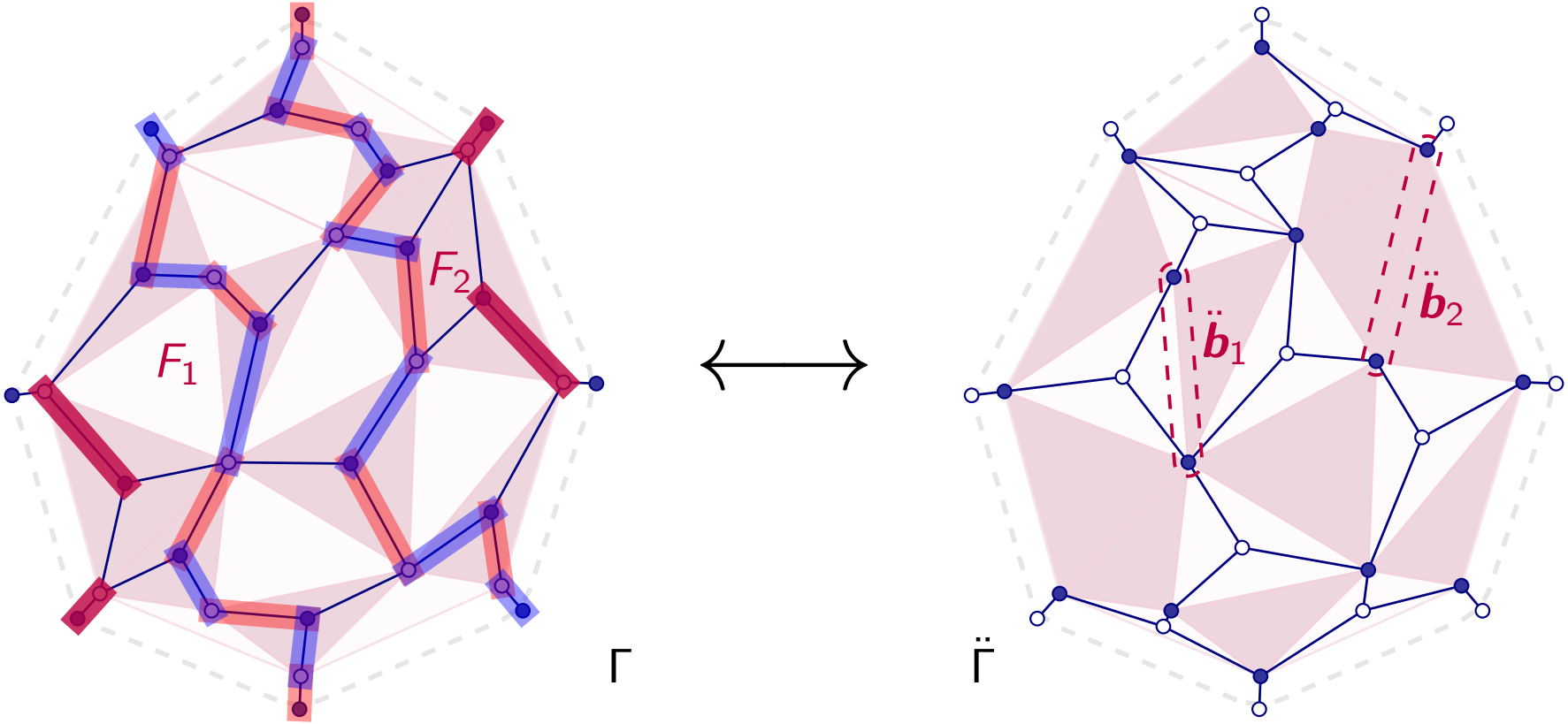

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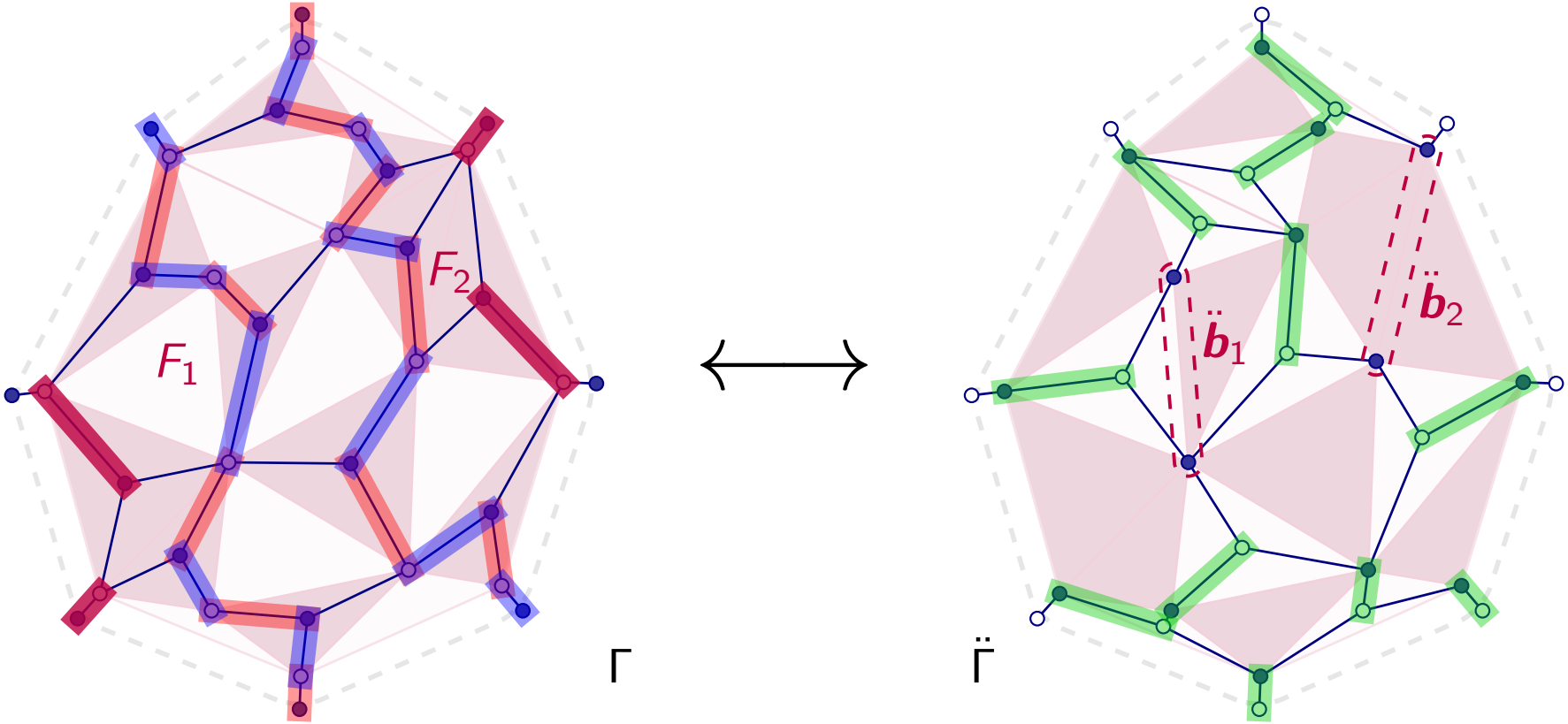

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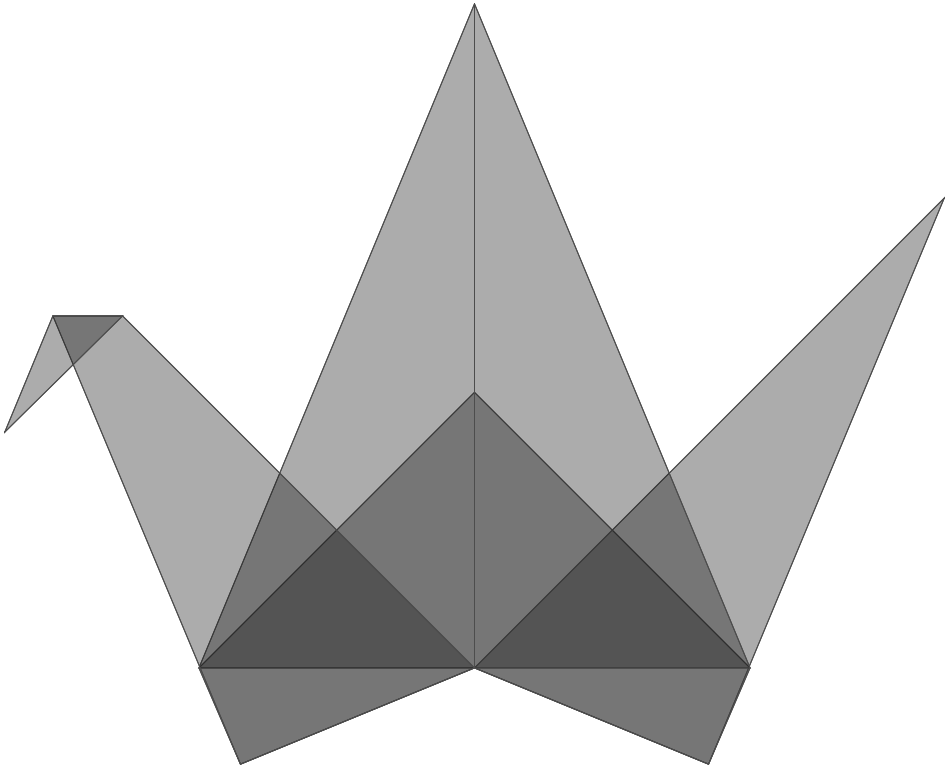

Thanks!