

Amplituhedra and Origami

Pavel Galashin (Cornell/UCLA)

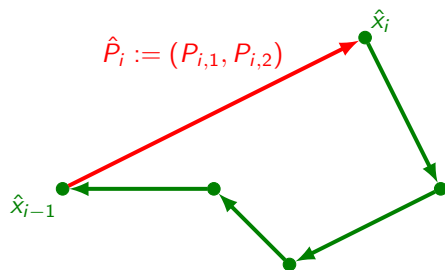
Stanford Mathematics Department Colloquium

January 22, 2026

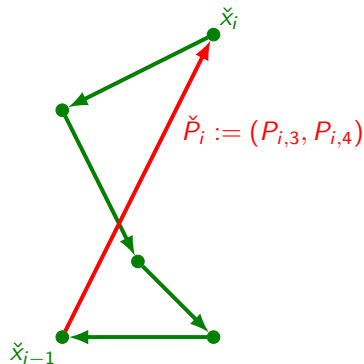
[arXiv:2410.09574](#), [arXiv:26+](#)

Overview

- Input:** $P_1, P_2, \dots, P_n \in \mathbb{R}^{2,2}$, **massless:** $P_i^2 := P_{i,1}^2 + P_{i,2}^2 - P_{i,3}^2 - P_{i,4}^2 = 0$,
momentum conservation: $P_1 + P_2 + \dots + P_n = 0$.



Space ($\mathbb{R}^2$)

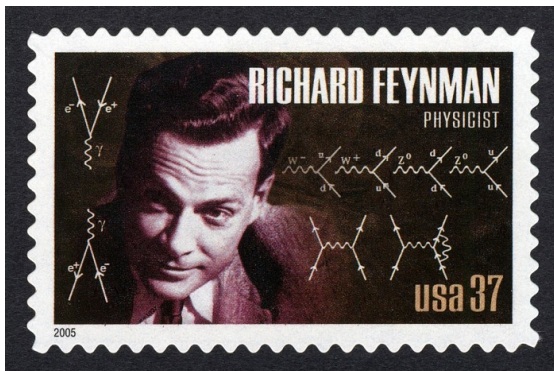


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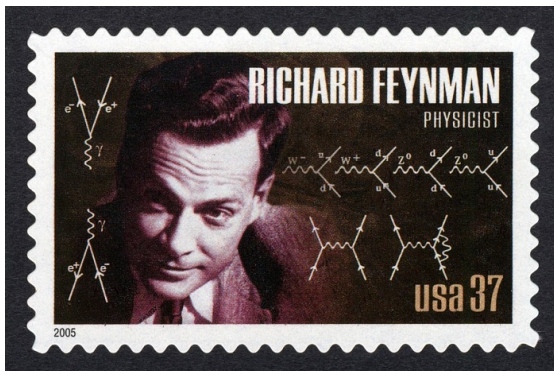


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- **Loop BCFW recursion:** $A^{(L)} = \sum_{\substack{L\text{-punctured planar} \\ \text{bicolored graph } \Gamma \in \Gamma_{k,n,(L)}^{\text{BCFW}}}} A_{\Gamma}^{(L)}.$

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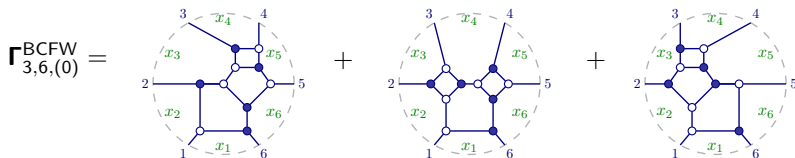
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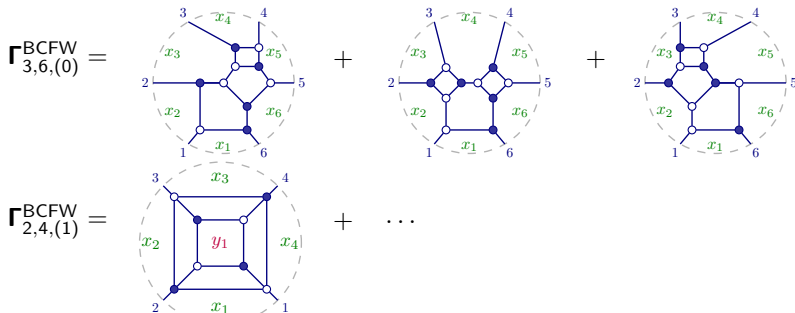
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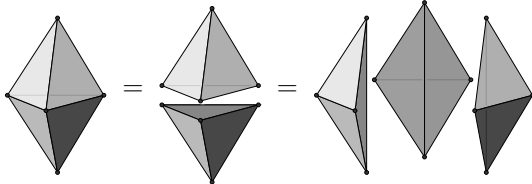
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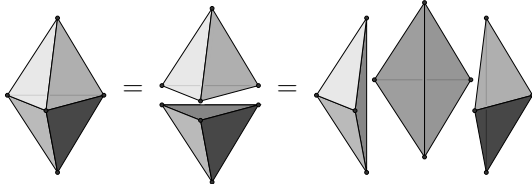


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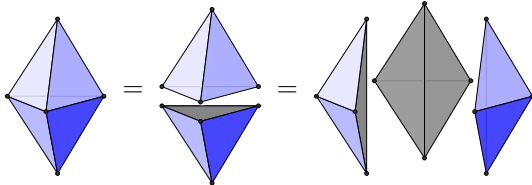


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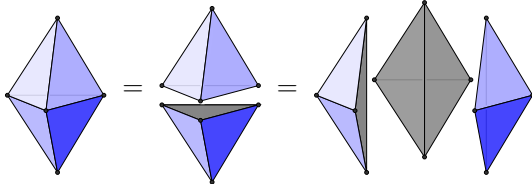


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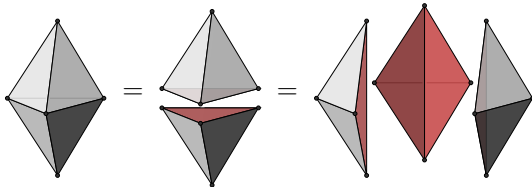


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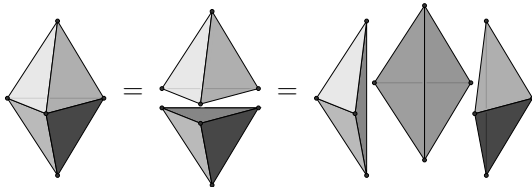


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$$\begin{aligned} \Gamma_{3,6,(0)}^{\text{BCFW}} &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \\ \Gamma_{2,4,(1)}^{\text{BCFW}} &= \text{Diagram 4} + \dots \end{aligned}$$

The diagrams illustrate the BCFW recursion. The first row shows three diagrams for $\Gamma_{3,6,(0)}^{\text{BCFW}}$, each a hexagon with internal lines and external legs labeled 1-6 and x_1 - x_6 . The second row shows a diagram for $\Gamma_{2,4,(1)}^{\text{BCFW}}$, which is a square with internal lines and external legs labeled 1-4 and x_1 - x_4 , with a red label y_1 in the center.

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Conjecture (BCFW triangulation conjecture [AHT'13])

The BCFW tiles triangulate $\mathcal{M}_{k,n}^{(L)}$ and $\mathcal{A}_{k-2,n}^{(L)}$.

Overview

- **Input:** $P_1, P_2, \dots, P_n \in \mathbb{R}^{2,2}$, massless: $P_i^2 := P_{i,1}^2 + P_{i,2}^2 - P_{i,3}^2 - P_{i,4}^2 = 0$, momentum conservation: $P_1 + P_2 + \dots + P_n = 0$.

- **Output:** Scattering amplitude

$$A(P_1, P_2, \dots, P_n) = \sum_{\substack{\text{Feynman} \\ \text{diagram } D}} A_D = A^{(L=0)} + A^{(L=1)} + A^{(L=2)} + \dots .$$

$L = |E| - |V| + 1$

- **Loop BCFW recursion:** $A^{(L)} = \sum A_{\Gamma}^{(L)}.$

[Britto–Cachazo–Feng–Witten '05] L -punctured planar
bicolored graph $\Gamma \in \Gamma_{k,n,(L)}^{\text{BCFW}}$

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Conjecture (BCFW triangulation conjecture [AHT'13])

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Theorem (G.'26+)

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- **Proof** relies on the origami–amplituhedron correspondence [G.'24].

≡ Kawasaki's theorem

🌐 6 languages ▾

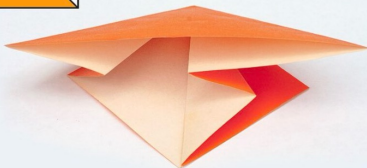
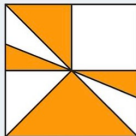
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In this example, the alternating sum of angles (clockwise from the bottom) is $90^\circ - 45^\circ + 22.5^\circ - 22.5^\circ + 45^\circ - 90^\circ + 22.5^\circ - 22.5^\circ = 0^\circ$. Since it adds to zero, the crease pattern may be flat-folded. 

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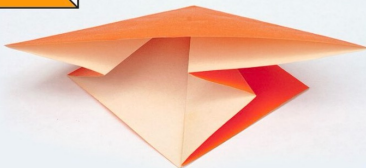
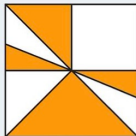
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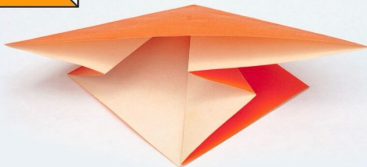
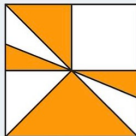
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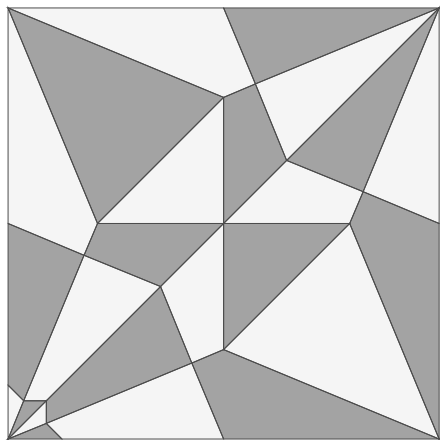


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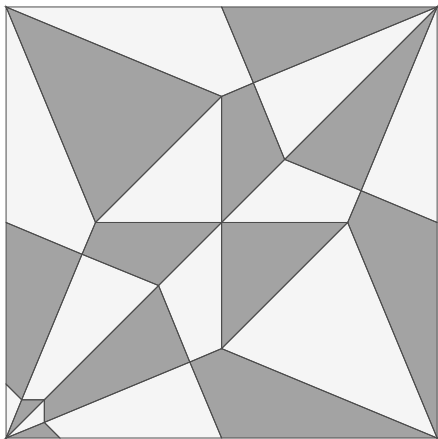


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Origami crease patterns (OCP)

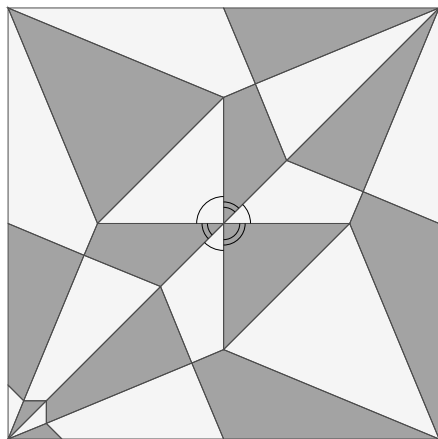


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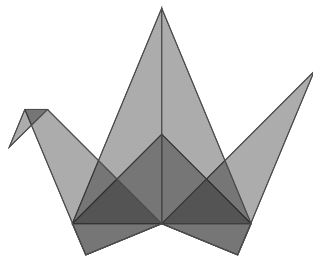
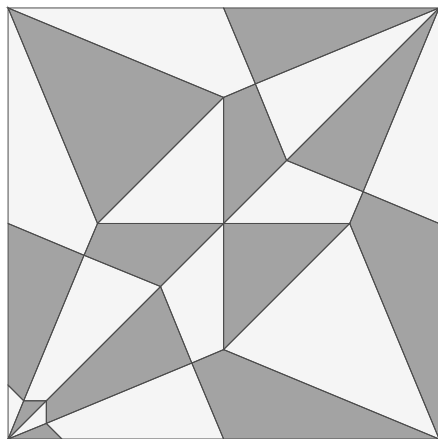
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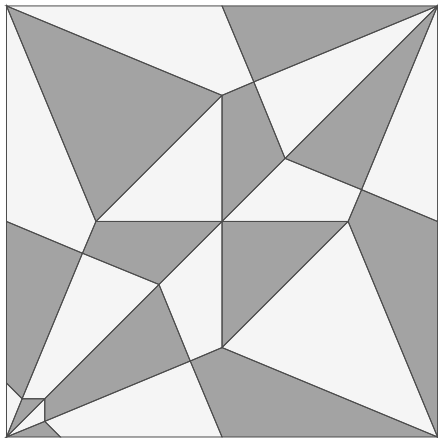


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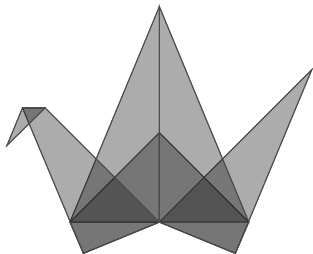
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- **Origami map** $\mathcal{O}$: isometry on each face preserving/reversing the orientations of white/black faces.

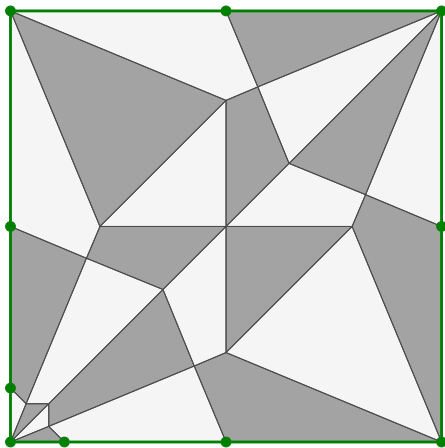


Space ($\mathbb{R}^2$)

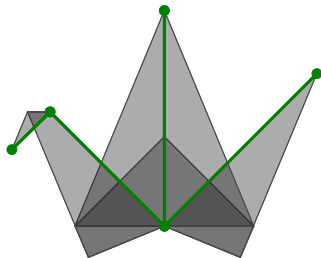


Time ($\mathbb{R}^2$)

- OCP + its origami folding = 2-dimensional discrete PL surface in $\mathbb{R}^{2,2}$.

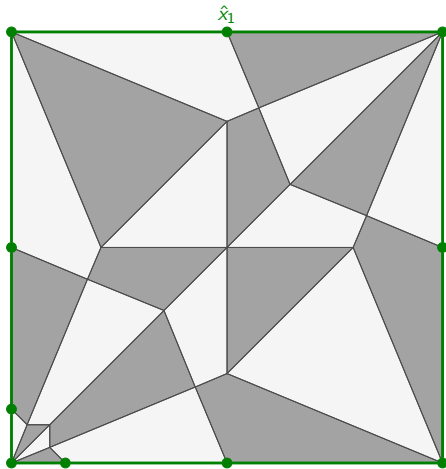


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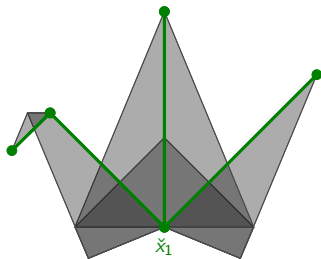


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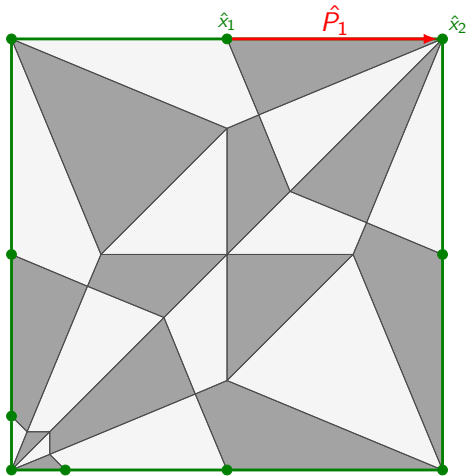


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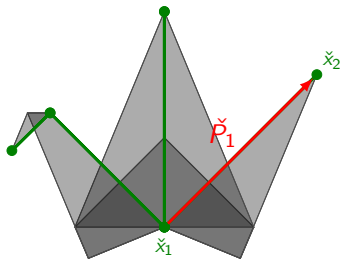


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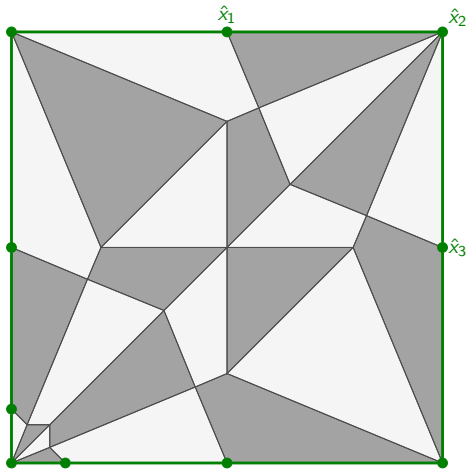


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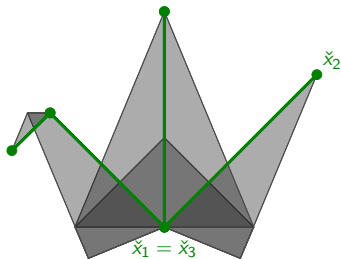


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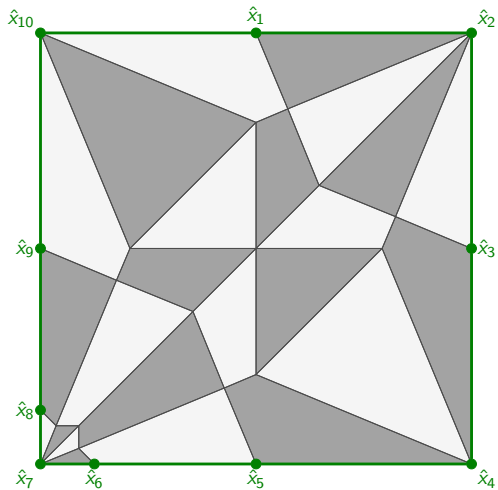


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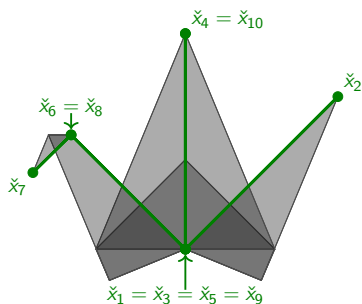


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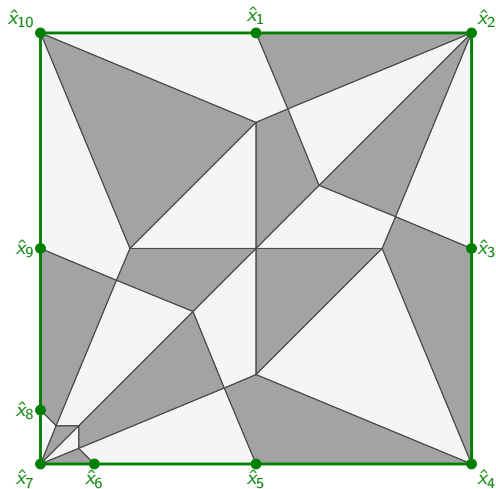


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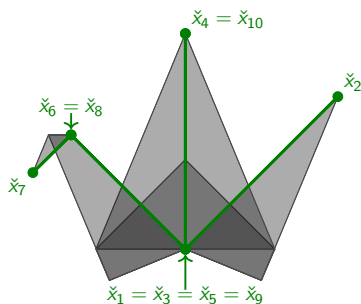


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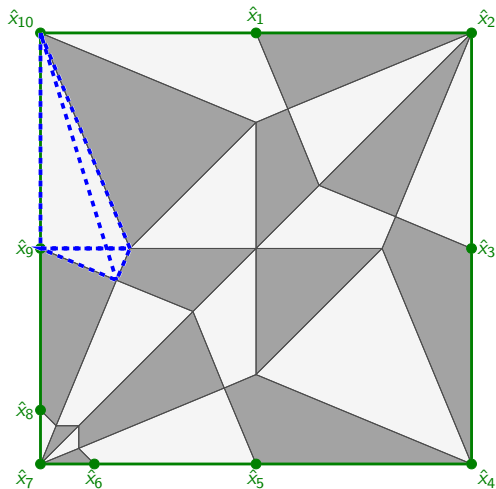


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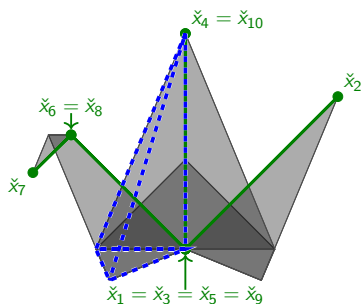


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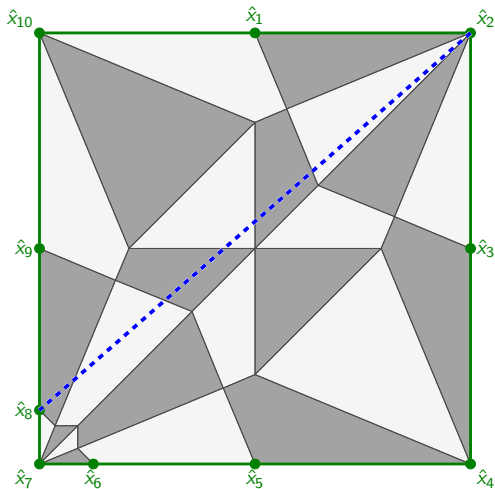


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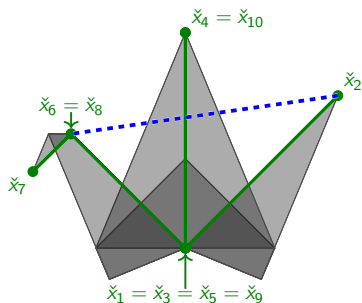


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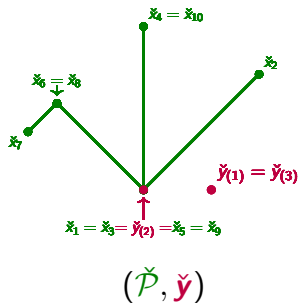
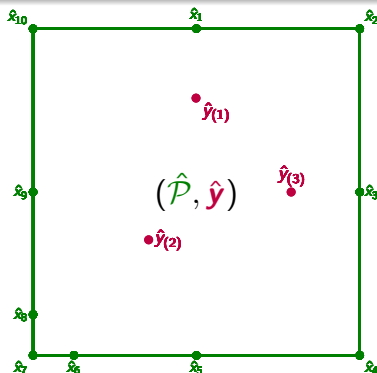
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Let $L \geq 0$. An L -punctured polygon is a pair $(\mathcal{P}, \mathbf{y})$, where

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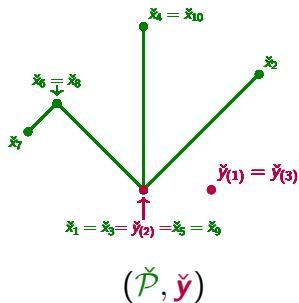
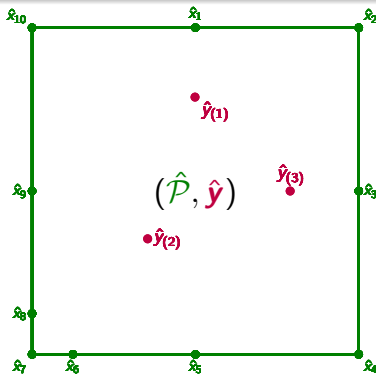
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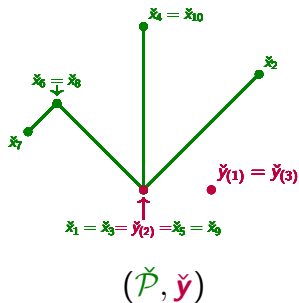
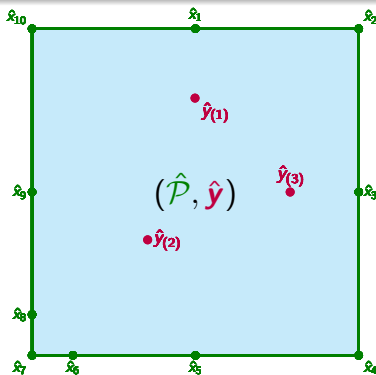
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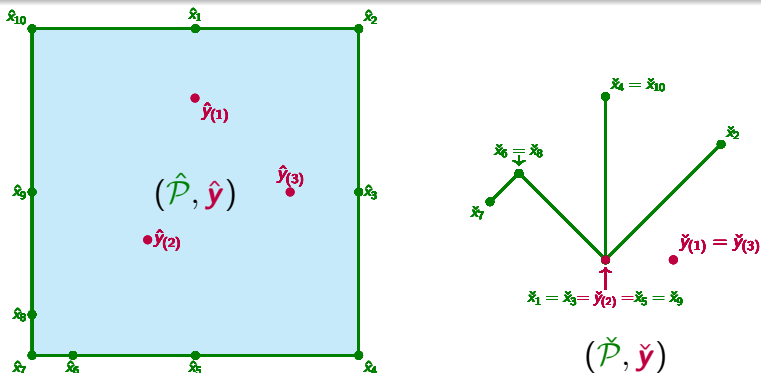
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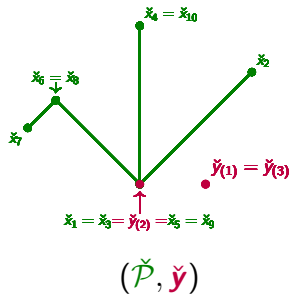
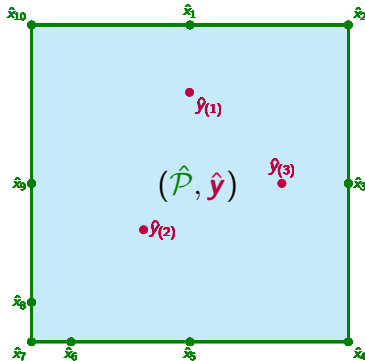
$$(x_i - x_{i-1})^2 = 0, \quad (x_i - x_j)^2 > 0, \quad (x_i - y_{(\rho)})^2 > 0, \quad (y_{(\rho)} - y_{(\gamma)})^2 > 0$$

for all non-adjacent $1 \leq i, j \leq n$ and all $1 \leq \rho \neq \gamma \leq L$, and such that

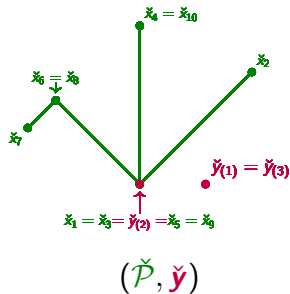
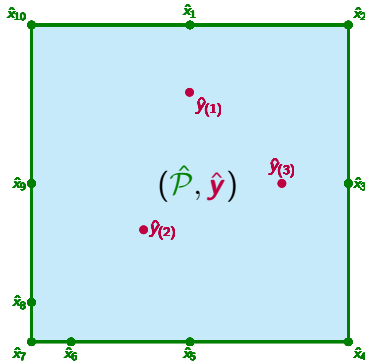
each $\hat{y}_{(\rho)}$ is located inside the polygon $\hat{\mathcal{P}} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$.



Problem: Find an OCP with boundary $\hat{\mathcal{P}}$ such that the origami map sends $\hat{x}_i \mapsto \check{x}_i$ and $\hat{y}_{(\rho)} \mapsto \check{y}_{(\rho)}$ for all i, ρ .



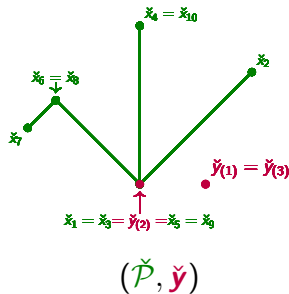
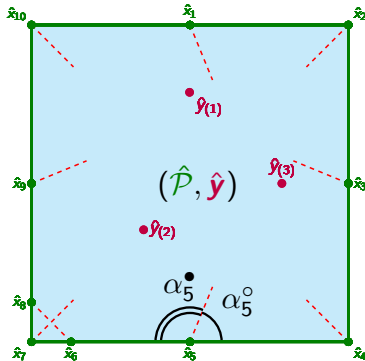
Problem: Find an OCP with boundary $\hat{p}$ such that the origami map sends $\hat{x}_i \mapsto \check{x}_i$ and $\hat{y}_{(\rho)} \mapsto \check{y}_{(\rho)}$ for all i, ρ .



Problem: Find an OCP with boundary $\hat{\mathcal{P}}$ such that the origami map sends $\hat{x}_i \mapsto \check{x}_i$ and $\hat{y}_{(\rho)} \mapsto \check{y}_{(\rho)}$ for all i, ρ .

- Recover white/black angle sums $(\alpha_i^\circ, \alpha_i^\bullet)$ from the geometry of $\mathcal{P}$:

$$\alpha_i^\circ + \alpha_i^\bullet = \hat{\alpha}_i \quad \text{and} \quad \alpha_i^\circ - \alpha_i^\bullet \equiv \check{\alpha}_i \pmod{2\pi}.$$

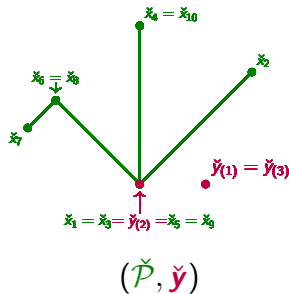
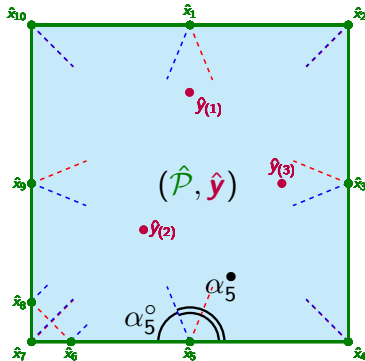


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- Red folding ray splits the angle $\hat{\alpha}_i$ into angles $(\alpha_i^\bullet, \alpha_i^\circ)$.

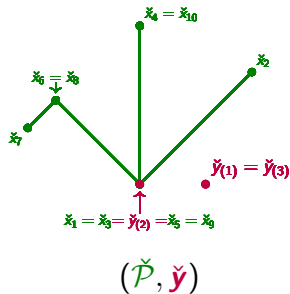
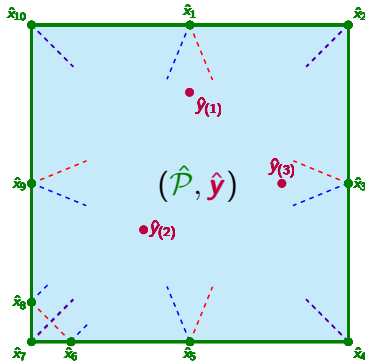


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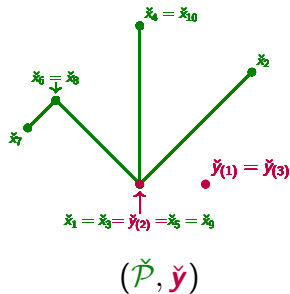
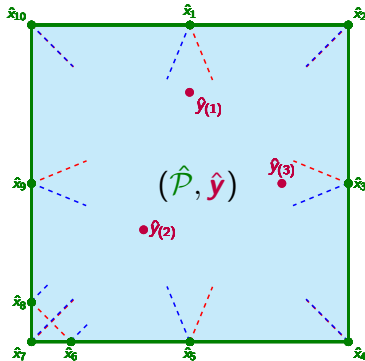
$$\alpha_i^\circ + \alpha_i^\bullet = \hat{\alpha}_i \quad \text{and} \quad \alpha_i^\circ - \alpha_i^\bullet \equiv \check{\alpha}_i \pmod{2\pi}.$$

- Red folding ray splits the angle $\hat{\alpha}_i$ into angles $(\alpha_i^\bullet, \alpha_i^\circ)$.
- Blue folding ray splits the angle $\hat{\alpha}_i$ into angles $(\alpha_i^\circ, \alpha_i^\bullet)$.



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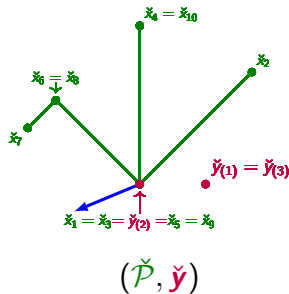
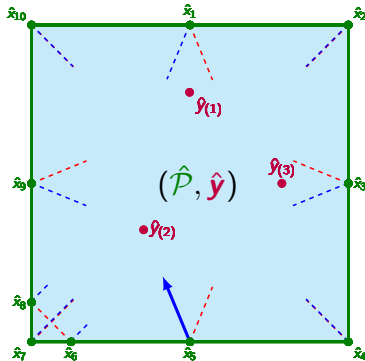
Origami Reconstruction Algorithm



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Origami Reconstruction Algorithm

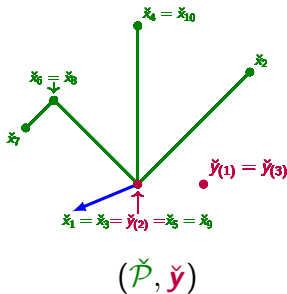
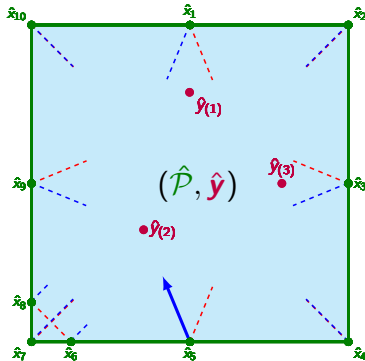
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Origami Reconstruction Algorithm

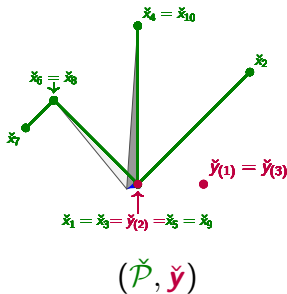
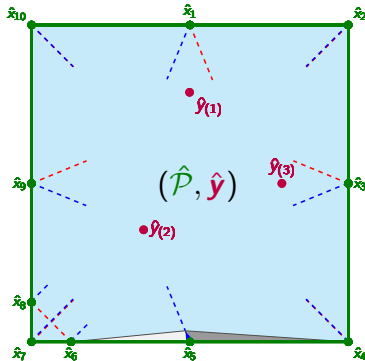
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Origami Reconstruction Algorithm

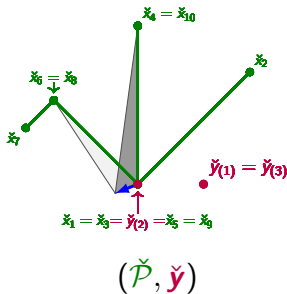
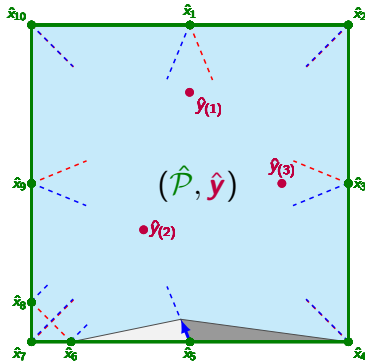
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Origami Reconstruction Algorithm

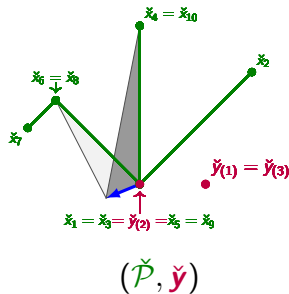
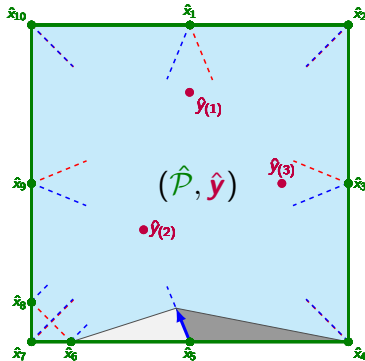
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Origami Reconstruction Algorithm

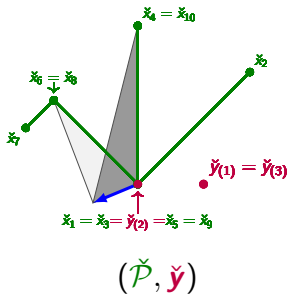
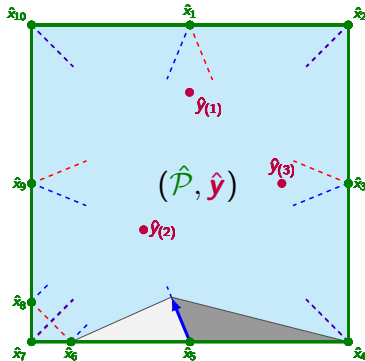
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Origami Reconstruction Algorithm

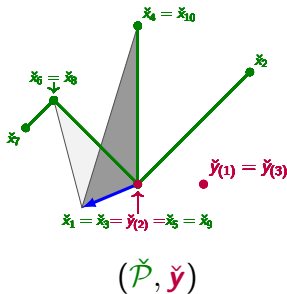
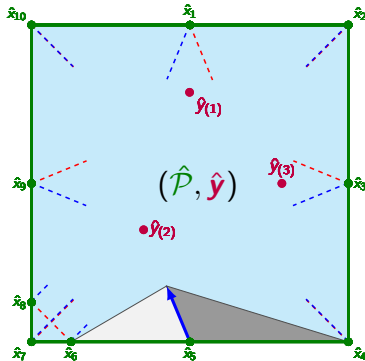
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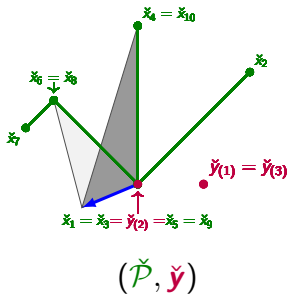
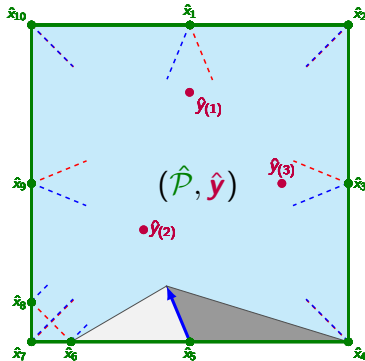
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Origami Reconstruction Algorithm

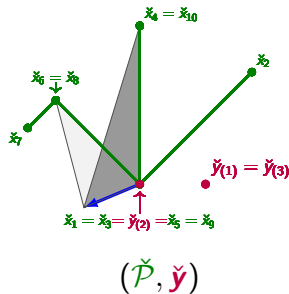
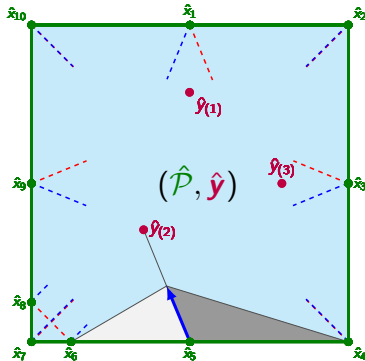
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Origami Reconstruction Algorithm

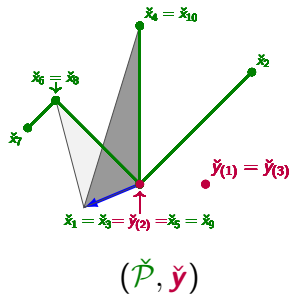
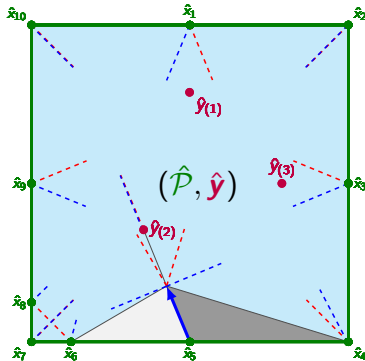
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Origami Reconstruction Algorithm

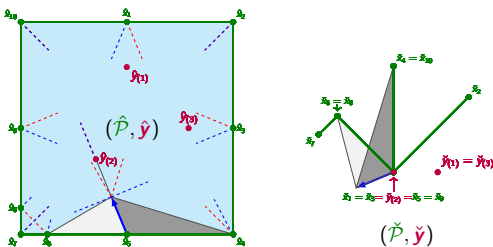
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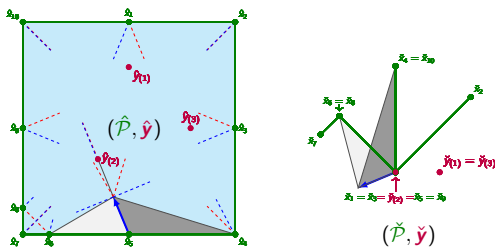
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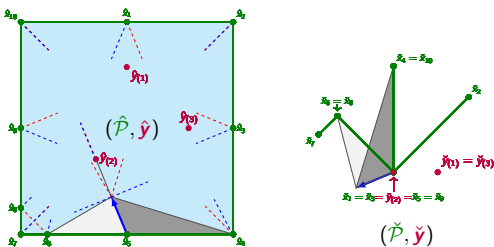
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Theorem (G.'26+)

The Origami Reconstruction Algorithm *works*.



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Origami Reconstruction Algorithm

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Theorem (G.'26+)

The Origami Reconstruction Algorithm works.

For all L -punctured polygons $(\mathcal{P}, \mathbf{y})$, it outputs a **valid, embedded** OCP.

- BCFW recursion $\rightarrow$ collection $\mathbf{\Gamma}_{k,n,(L)}^{\text{BCFW}}$ of L -punctured planar bicolored graphs.

$$\begin{array}{c} \text{Diagram 1: A circle with 8 external legs labeled } x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \text{ in green. Inside is a light blue oval labeled } \mathbf{\Gamma}_{k,n,(L)}^{\text{BCFW}}. \end{array} = \sum_{\substack{k_1+k_2=k+1 \\ n_1+n_2=n+2 \\ L_1+L_2=L}} \left(\begin{array}{c} \text{Diagram 2: Two light blue ovals, } \mathbf{\Gamma}_{k_1,n_1,(L_1)}^{\text{BCFW}} \text{ and } \mathbf{\Gamma}_{k_2,n_2,(L_2)}^{\text{BCFW}}, \text{ connected by a horizontal line with a white circle on the left and a black dot on the right.} \end{array} + \begin{array}{c} \text{Diagram 3: A light blue oval labeled } \mathbf{\Gamma}_{k+1,n+2,(L-1)}^{\text{BCFW}} \text{ with a red arc labeled } \gamma(\rho) \text{ connecting the bottom legs } x_1 \text{ and } x_3. \end{array} \right)$$

- BCFW recursion $\rightarrow$ collection $\mathbb{R}_{k,n,(L)}^{\text{BCFW}}$ of L -punctured planar bicolored graphs.

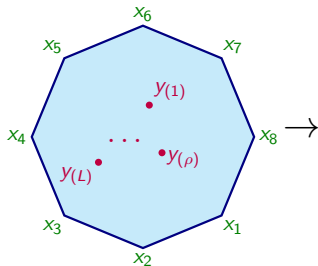
$$\mathbb{R}_{k,n,(L)}^{\text{BCFW}} = \sum_{\substack{k_1+k_2=k+1 \\ n_1+n_2=n+2 \\ L_1+L_2=L}} \mathbb{R}_{k_1,n_1,(L_1)}^{\text{BCFW}} + \mathbb{R}_{k+1,n+2,(L-1)}^{\text{BCFW}}$$

- Planar dual pictures = possibilities in the Origami Reconstruction Algorithm:

- BCFW recursion $\rightarrow$ collection $\mathbf{\Gamma}_{k,n,(L)}^{\text{BCFW}}$ of L -punctured planar bicolored graphs.

$$\begin{array}{c} \text{Diagram 1: A circle with 8 external vertices labeled } x_1 \text{ through } x_8 \text{ in green. Inside is a light blue oval labeled } \mathbf{\Gamma}_{k,n,(L)}^{\text{BCFW}}. \end{array} = \sum_{\substack{k_1+k_2=k+1 \\ n_1+n_2=n+2 \\ L_1+L_2=L}} \begin{array}{c} \text{Diagram 2: A circle with 8 external vertices } x_1 \text{ through } x_8. \text{ Inside are two light blue ovals: } \mathbf{\Gamma}_{k_1,n_1,(L_1)}^{\text{BCFW}} \text{ and } \mathbf{\Gamma}_{k_2,n_2,(L_2)}^{\text{BCFW}}. \text{ They are connected by a horizontal edge with a white circle on the left and a blue circle on the right.} \end{array} + \begin{array}{c} \text{Diagram 3: A circle with 8 external vertices } x_1 \text{ through } x_8. \text{ Inside is a light blue oval labeled } \mathbf{\Gamma}_{k+1,n+2,(L-1)}^{\text{BCFW}}. \text{ A red arc labeled } y(\rho) \text{ connects the two vertices on the bottom edge (} x_2 \text{ and } x_1 \text{).} \end{array}$$

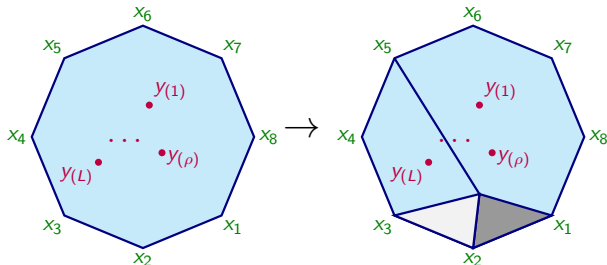
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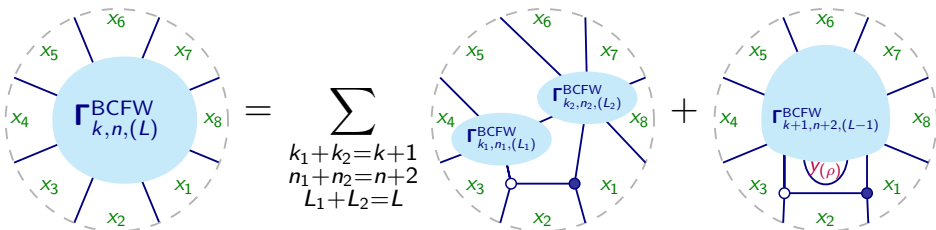
- BCFW recursion $\rightarrow$ collection $\Gamma_{k,n,(L)}^{\text{BCFW}}$ of L -punctured planar bicolored graphs.

$$\Gamma_{k,n,(L)}^{\text{BCFW}} = \sum_{\substack{k_1+k_2=k+1 \\ n_1+n_2=n+2 \\ L_1+L_2=L}} \Gamma_{k_1,n_1,(L_1)}^{\text{BCFW}} \Gamma_{k_2,n_2,(L_2)}^{\text{BCFW}} + \Gamma_{k+1,n+2,(L-1)}^{\text{BCFW}}$$

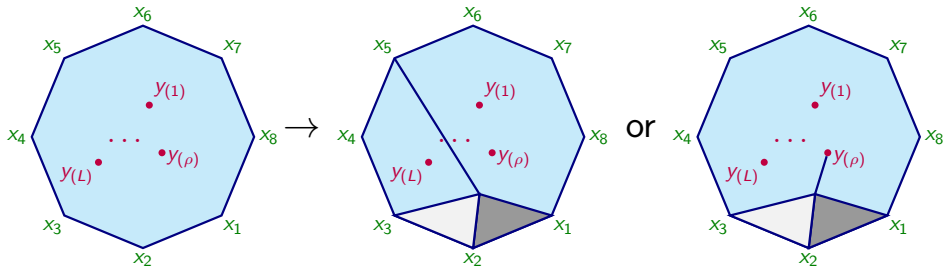
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- BCFW recursion $\rightarrow$ collection $\Gamma_{k,n,(L)}^{\text{BCFW}}$ of L -punctured planar bicolored graphs.



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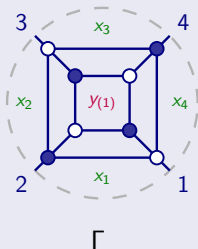


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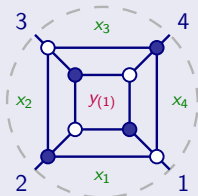


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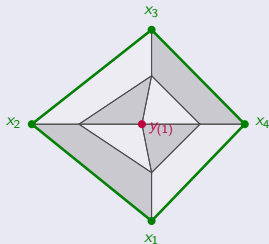
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Γ



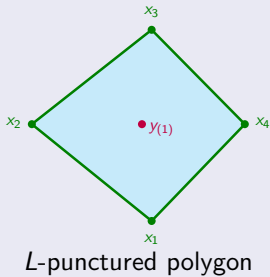
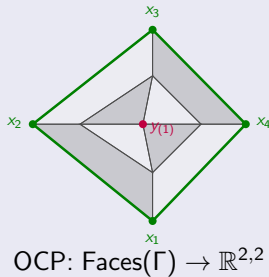
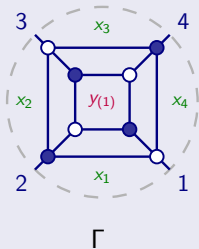
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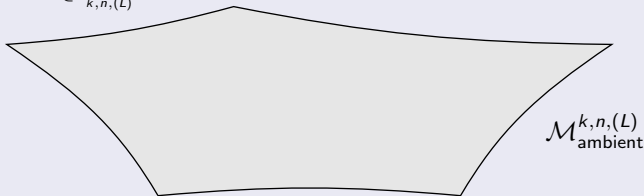


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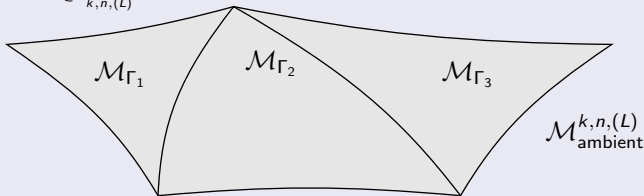


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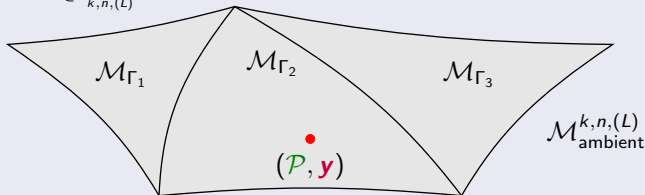


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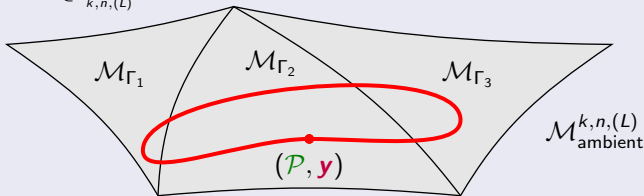


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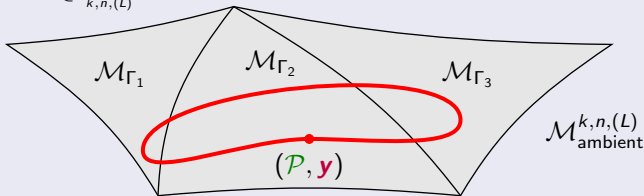


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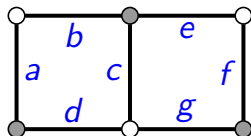


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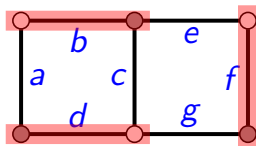
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- **Dimer model**: study of perfect matchings on weighted bipartite graphs



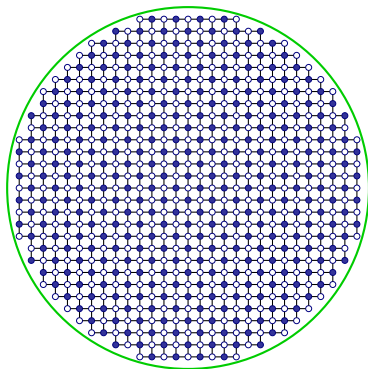
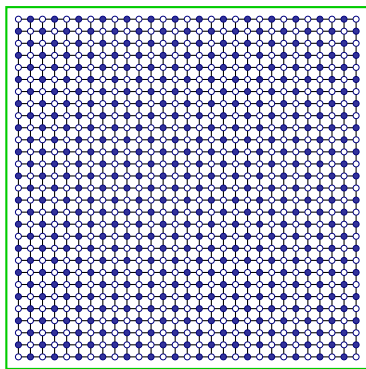
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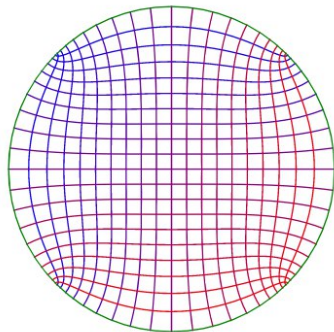
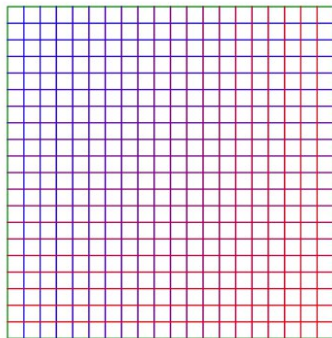
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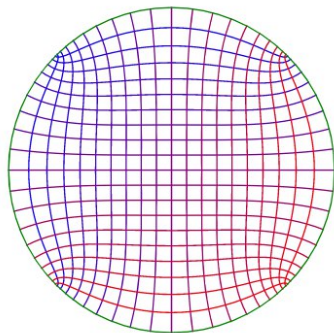
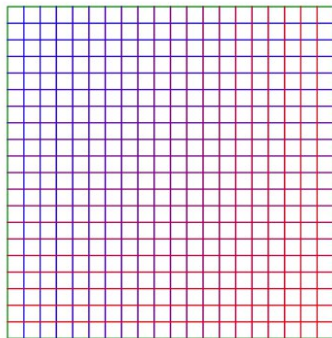
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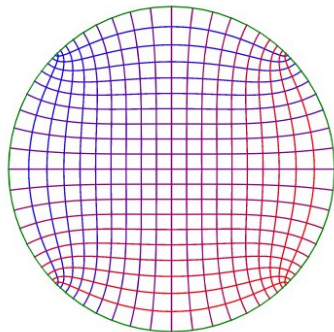
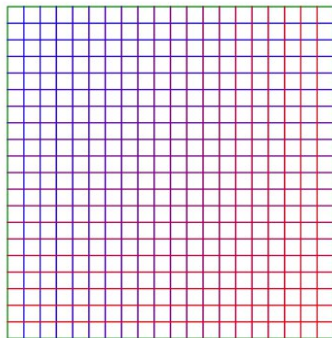
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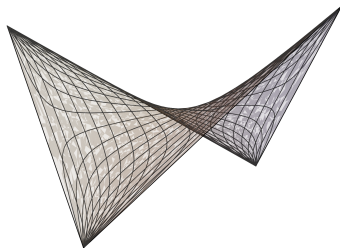
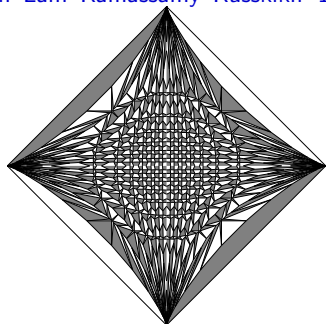
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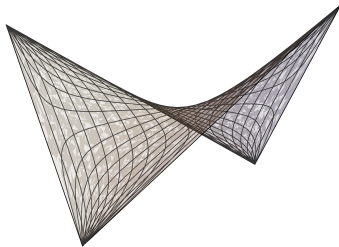
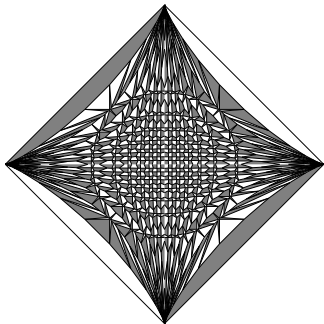
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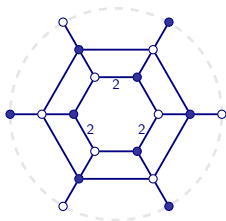
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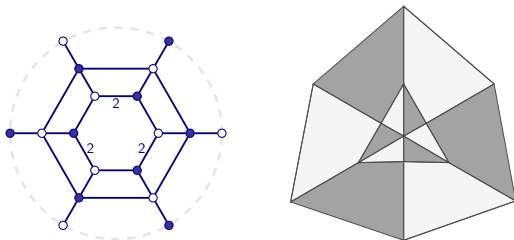
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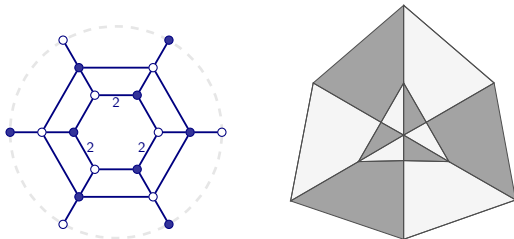
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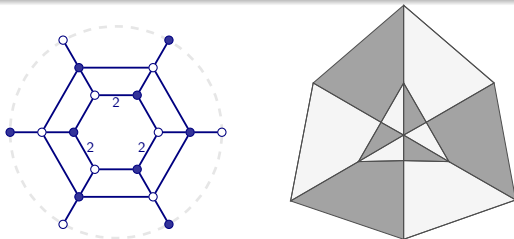
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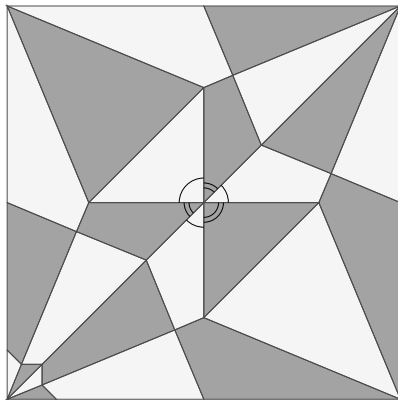
Proof.

Follows from the origami–amplituhedron correspondence. □

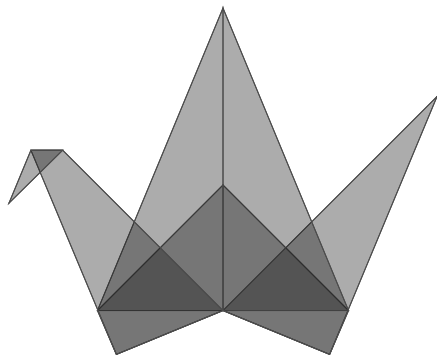


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- Convergence results for dimer model observables on OCPs: [KLRR '18], [CLR '21]
- OCPs: embedded in the plane, satisfy angle condition: $\hat{\alpha}^\bullet = \hat{\alpha}^\circ = \pi$
- Kenyon–Smirnov primitives: not necessarily embedded, $\hat{\alpha}^\bullet \equiv \hat{\alpha}^\circ \equiv \pi \pmod{2\pi}$

Theorem (Origami–Amplituhedron Correspondence (G. 2024))

A Kenyon–Smirnov primitive gives rise to an (embedded) OCP $\iff$ its $\mathbb{R}^{2,2}$ -boundary polygon belongs to the momentum amplituhedron $\mathcal{M}_{k,n}^{(L=0)}$.

- Proofs of BCFW Triangulation Conjecture of [AHT '13] and of the OCP Existence Conjecture of [KLRR '18], [CLR '21] follow from this theorem.

- Conformal invariance for the dimer model on $\mathbb{Z}^2$ [Kenyon '00]
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Conjecture (BCFW Triangulation Conjecture [AHT '13])

The BCFW cells triangulate the amplituhedron.

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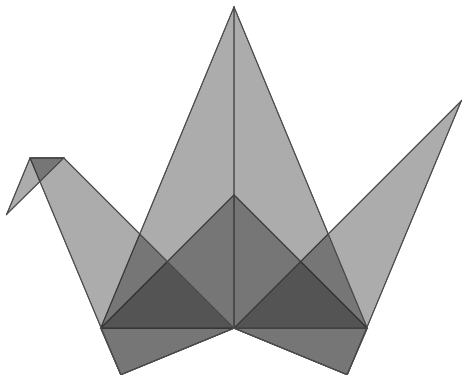
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Conjecture (OCP Existence Conjecture [KLRR '18], [CLR '21])

Every weighted planar bipartite graph Γ can be realized geometrically as a planar dual of an embedded OCP.



Thanks!