Zamolodchikov periodicity and integrability

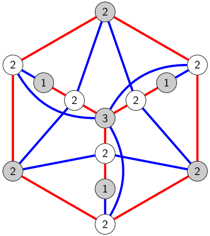
Pavel Galashin

MIT

galashin@mit.edu

University of Michigan, September 15, 2017

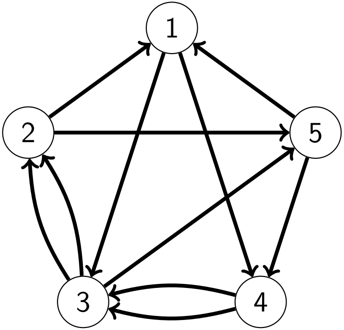
Joint work with Pavlo Pylyavskyy



Part 1: T-systems

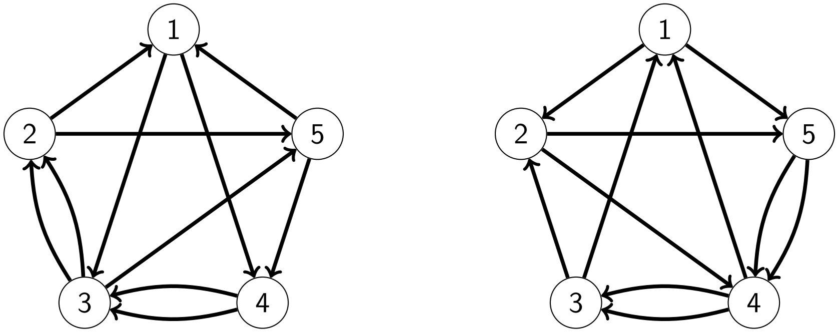
General T-systems (Nakanishi, 2011)

Q



General T-systems (Nakanishi, 2011)

Q $\qquad\qquad$ $\mu_1(Q)$



General T-systems (Nakanishi, 2011)

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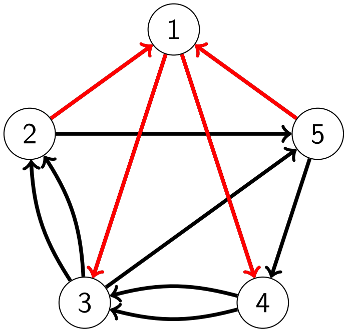
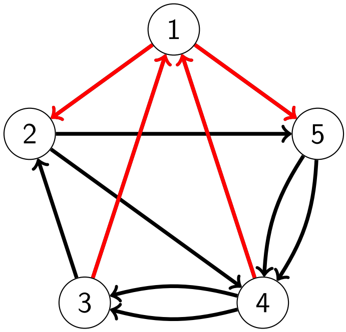


$\mu_1(Q)$



General T-systems (Nakanishi, 2011)

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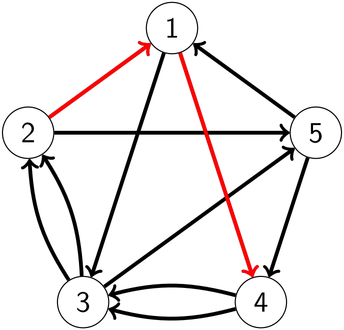

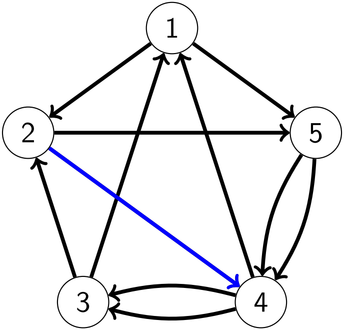

General T-systems (Nakanishi, 2011)

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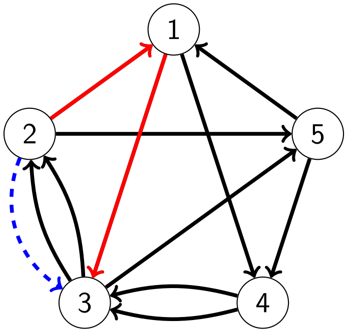
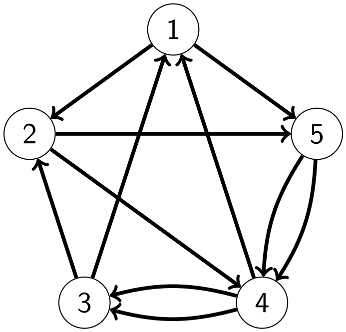


$\mu_1(Q)$



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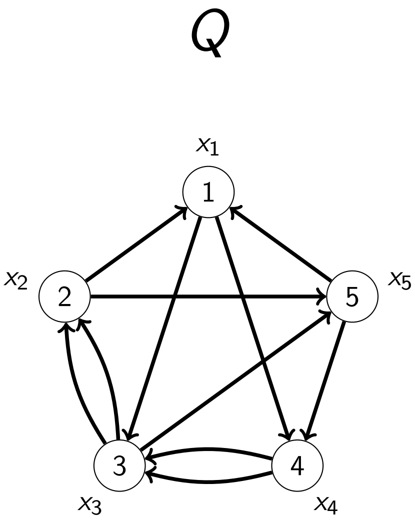
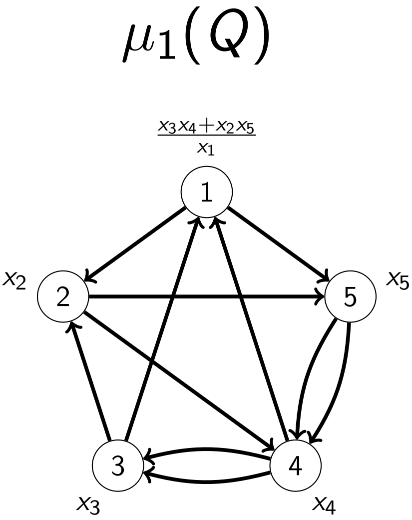
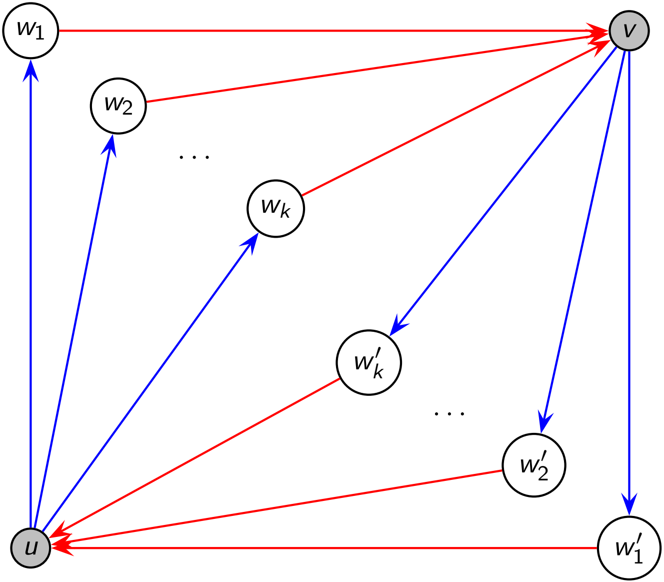
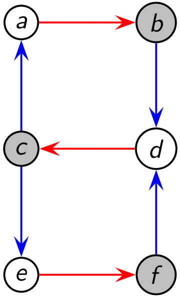




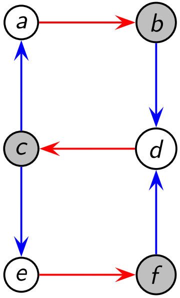
Bipartite **recurrent** quivers



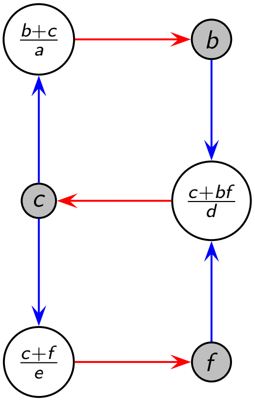
Bipartite T-system



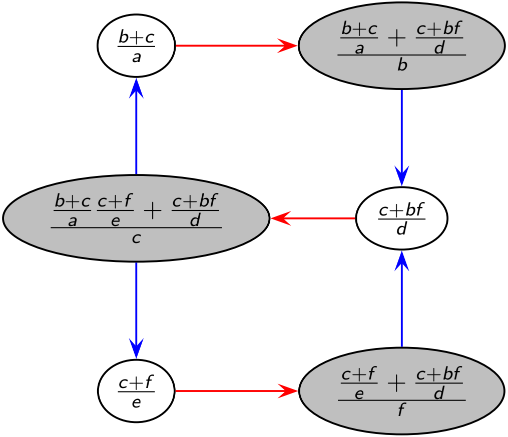
Bipartite T-system

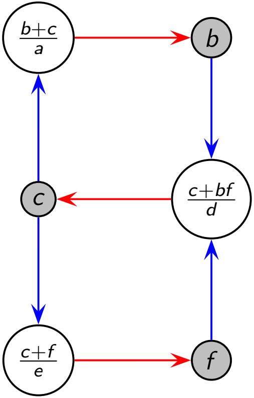


$\longrightarrow$

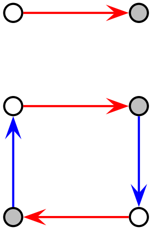


Bipartite T-system

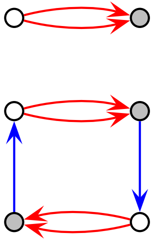




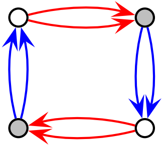
Four classes of quivers



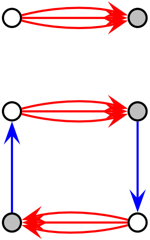
“finite $\boxtimes$ finite”



“affine $\boxtimes$ finite”

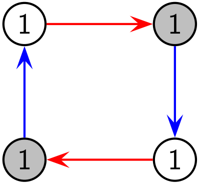


“affine $\boxtimes$ affine”



“wild”

Example: finite ⊠ finite



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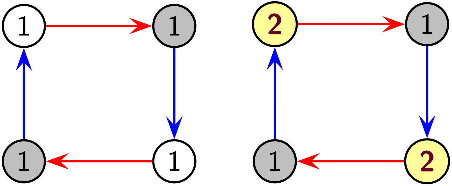

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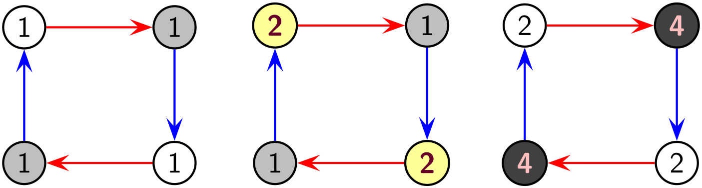

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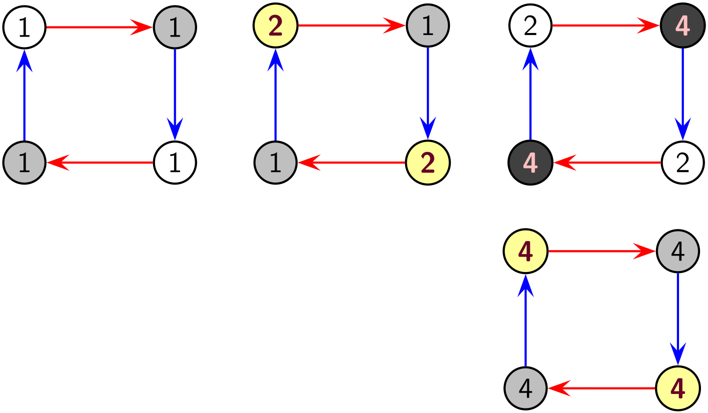

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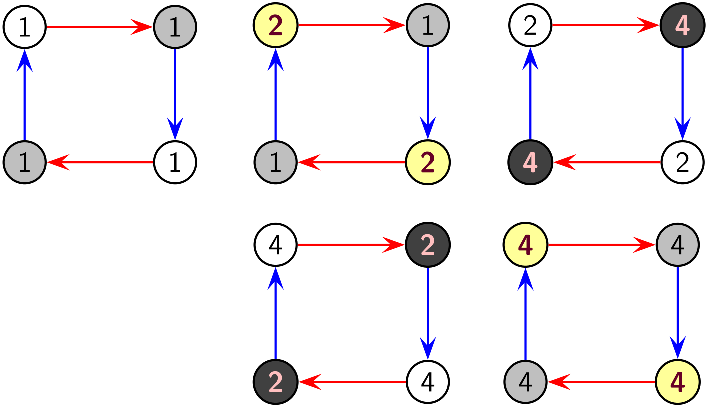

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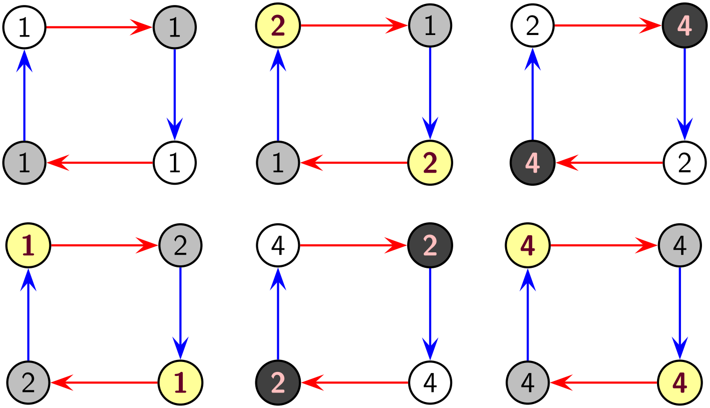
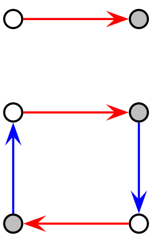
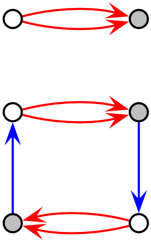


Four classes of quivers

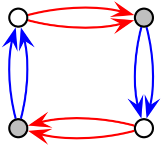


“finite ⊠ finite”

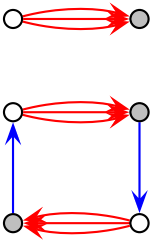
periodic



“affine ⊠ finite”



“affine ⊠ affine”



“wild”

Example: affine ⊠ finite

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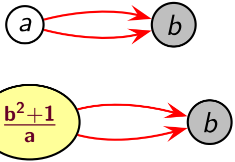

Example: affine ⊠ finite

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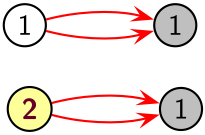
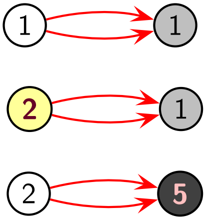


Example: affine ⊠ finite



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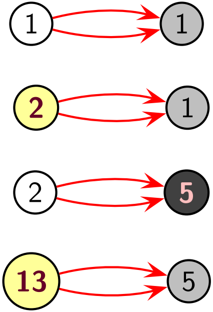

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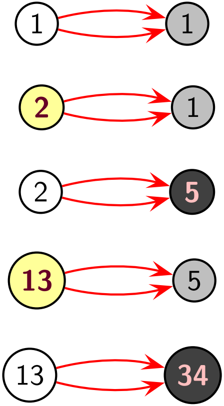

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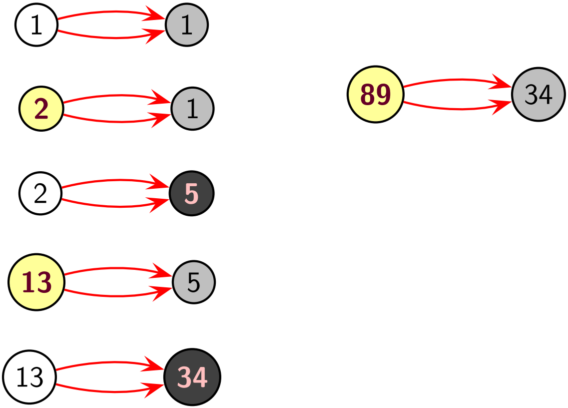

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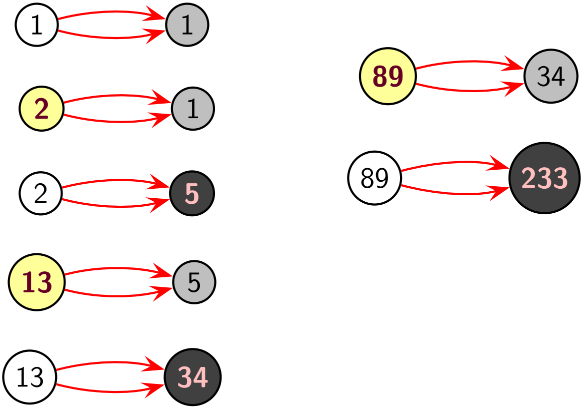

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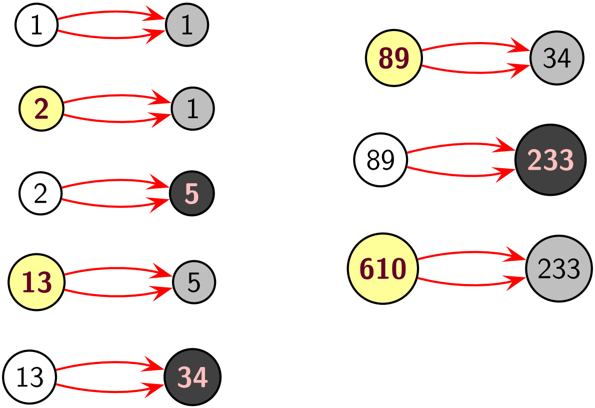

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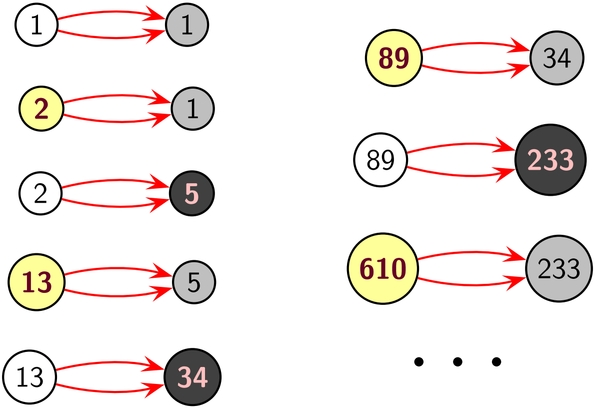

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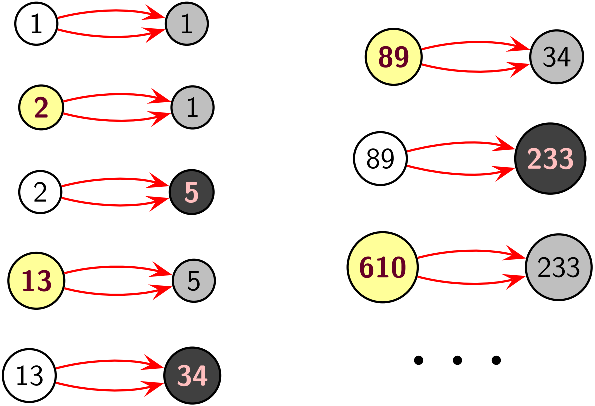
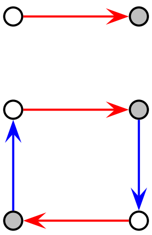


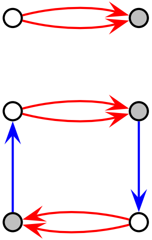
$$x_{n+1} = 3x_n - x_{n-1}$$

Four classes of quivers



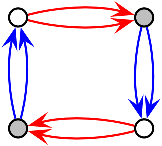
"finite $\boxtimes$ finite"

periodic

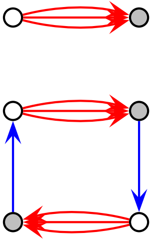


"affine $\boxtimes$ finite"

linearizable



"affine $\boxtimes$ affine"



"wild"

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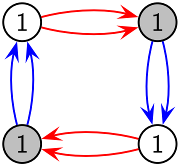
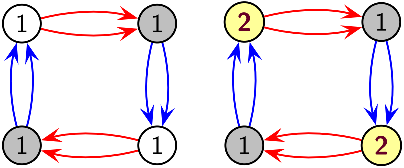


Example: affine ⊠ affine



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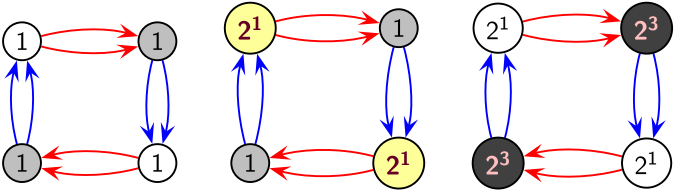

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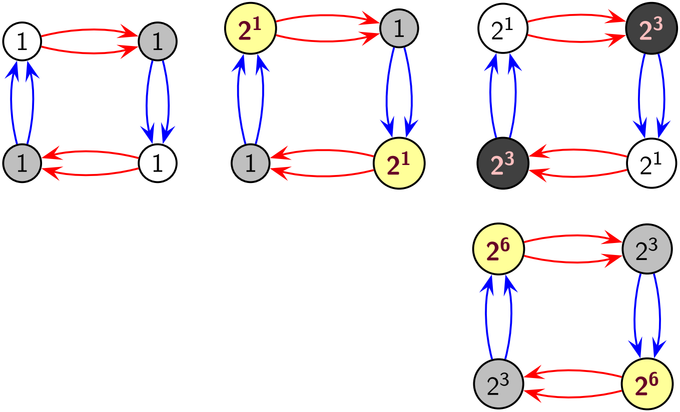

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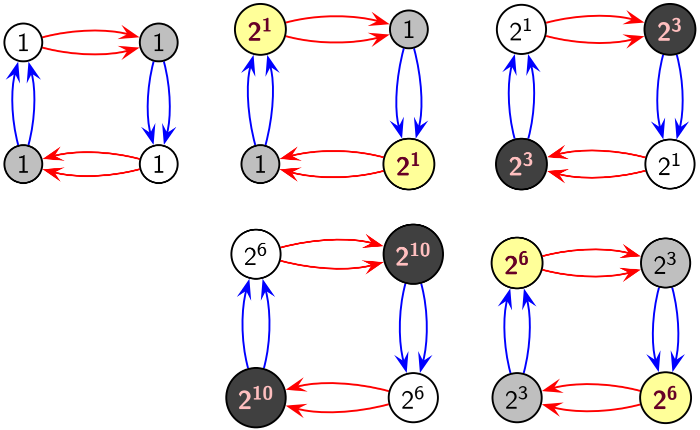

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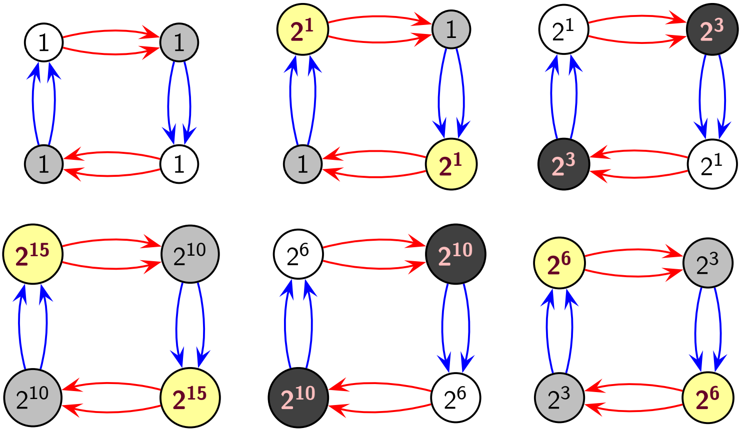

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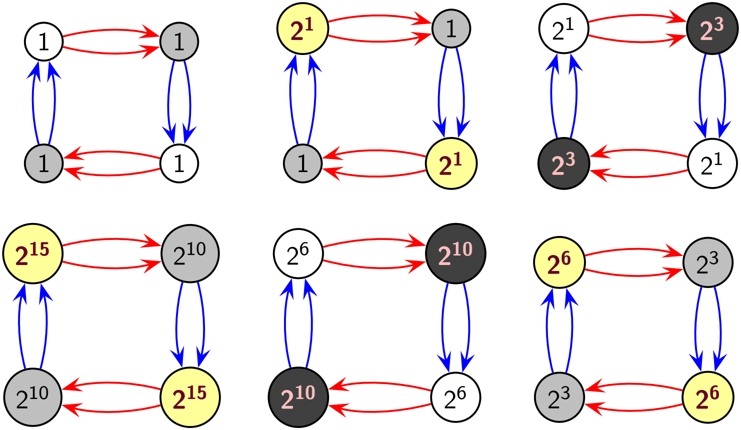
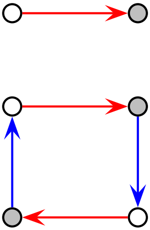


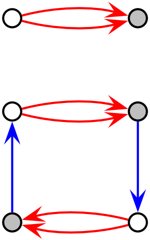
$$2^{\binom{n}{2}}$$

Four classes of quivers



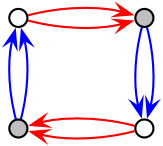
"finite ⊠ finite"

periodic



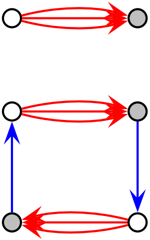
"affine ⊠ finite"

linearizable



"affine ⊠ affine"

grows as $\exp(t^2)$



"wild"

Example: wild

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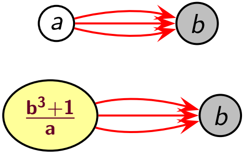

Example: wild

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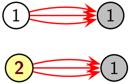

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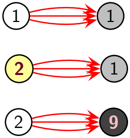

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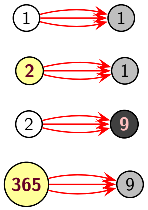

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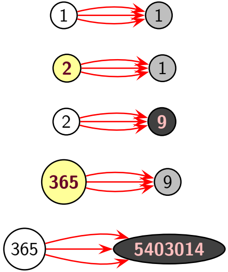

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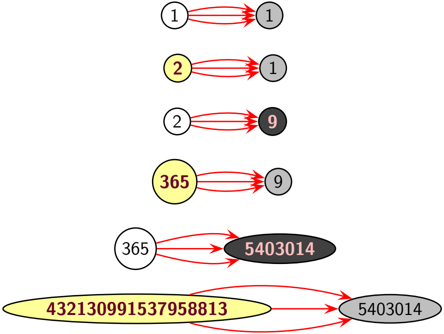

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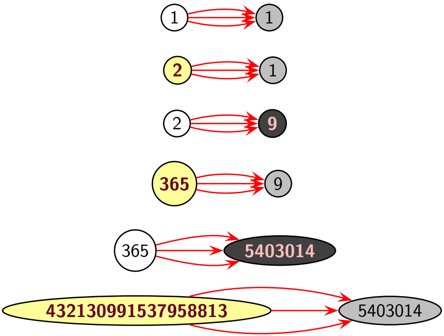


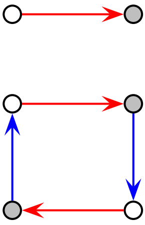
A003818 a(1)=a(2)=1, a(n+1) = (a(n)^3 +1)/a(n-1).

1, 1, 2, 9, 365, 5403014, 432130991537958813, 14935169284101525874491673463268414536523593057 (list; graph; refs; lis

OFFSET 1,3

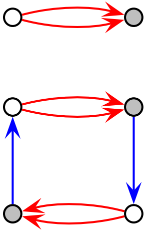
COMMENTS The term a(9) has 121 digits. - Harvey P. Dale,

Four classes of quivers



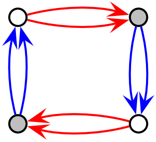
"finite ⊠ finite"

periodic



"affine ⊠ finite"

linearizable



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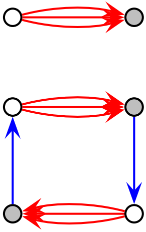
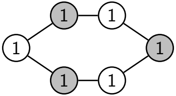
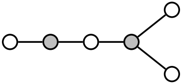
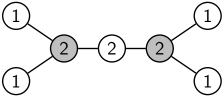
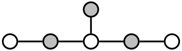
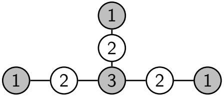
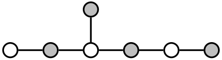
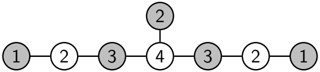
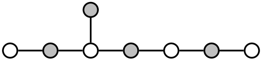
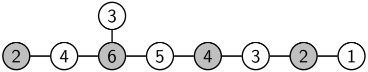


"wild"

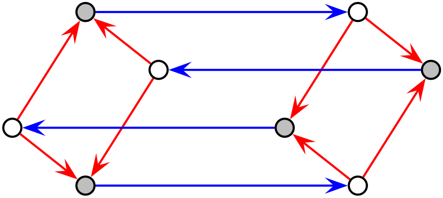
grows as $\exp(\exp(t))$

Part 2: The master conjecture

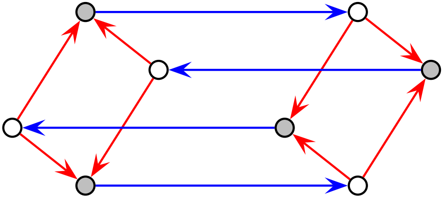
ADE Dynkin diagrams

Name	Finite diagram	Affine diagram	Name
A_n			$\hat{A}_{n-1}$
D_n			$\hat{D}_{n-1}$
E_6			$\hat{E}_6$
E_7			$\hat{E}_7$
E_8			$\hat{E}_8$

Affine $\boxtimes$ finite quivers

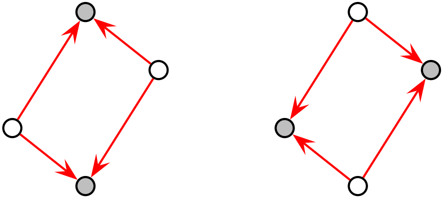


Affine $\boxtimes$ finite quivers



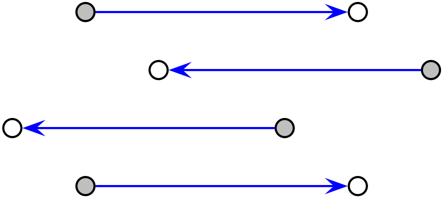
- Bipartite recurrent quiver

Affine $\boxtimes$ finite quivers



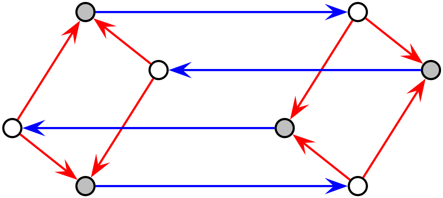
- Bipartite recurrent quiver
- All red components are **affine** Dynkin diagrams

Affine $\boxtimes$ finite quivers



- Bipartite recurrent quiver
- All red components are **affine** Dynkin diagrams
- All blue components are **finite** Dynkin diagrams

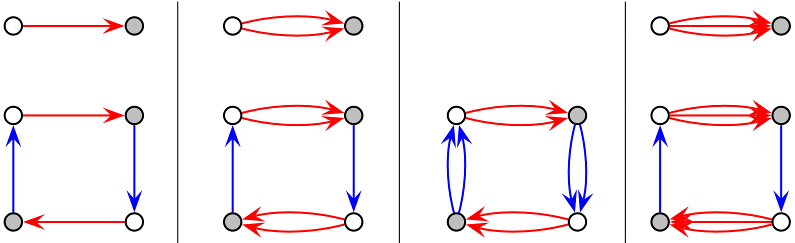
Affine ⊠ finite quivers



- Bipartite recurrent quiver
- All red components are **affine** Dynkin diagrams
- All blue components are **finite** Dynkin diagrams

"**Affine** ⊠ **finite quiver**"

Four classes of quivers



“finite ⊠ finite” — periodic

“affine ⊠ finite” — linearizable

“affine ⊠ affine” — grows as $\exp(t^2)$

“wild” — grows as $\exp(\exp(t))$

Master conjecture

Conjecture (G.-Pylyavskyy, 2016)

Master conjecture

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- *finite* $\boxtimes$ *finite* $\iff$ *periodic*

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Conjecture (G.-Pylyavskyy, 2016)

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Master conjecture

Conjecture (G.-Pylyavskyy, 2016)

- *finite* $\boxtimes$ *finite* $\Longleftrightarrow$ *periodic*
- *affine* $\boxtimes$ *finite* $\Longleftrightarrow$ *linearizable, but not periodic*
- *affine* $\boxtimes$ *affine* $\Longleftrightarrow$ *grows as* $\exp(t^2)$

Master conjecture

Conjecture (G.-Pylyavskyy, 2016)

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- *affine* $\boxtimes$ *affine* $\iff$ *grows as* $\exp(t^2)$
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Results

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Theorem (G.-Pylyavskyy, 2016)

Periodic $\iff$ *finite* $\boxtimes$ *finite*

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Linearizable $\implies$ *affine* $\boxtimes$ *finite or finite* $\boxtimes$ *finite*

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Theorem (G.-Pylyavskyy, 2017)

Grows slower than $\exp(\exp(t)) \implies$ *affine* $\boxtimes$ *affine, affine* $\boxtimes$ *finite, or finite* $\boxtimes$ *finite*

Results

Theorem (G.-Pylyavskyy, 2016)

Periodic $\Longleftrightarrow$ *finite* $\boxtimes$ *finite*

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Theorem (G.-Pylyavskyy, 2017)

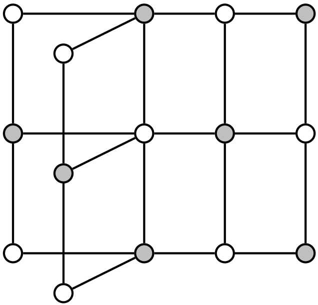
Grows slower than $\exp(\exp(t))$ $\Longrightarrow$ *affine* $\boxtimes$ *affine, affine* $\boxtimes$ *finite, or finite* $\boxtimes$ *finite*

What is left:

Conjecture (G.-Pylyavskyy, 2017)

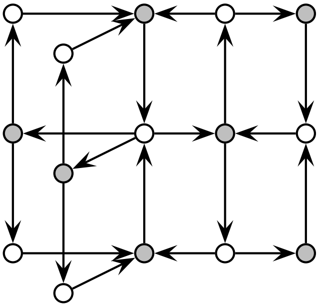
- *affine* $\boxtimes$ *finite* $\Longrightarrow$ *linearizable*
- *affine* $\boxtimes$ *affine* $\Longrightarrow$ *grows as* $\exp(t^2)$

Tensor product



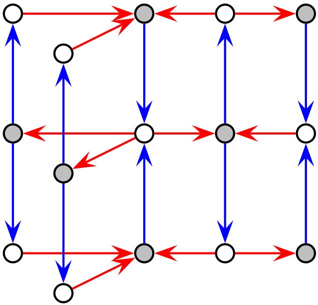
$D_5 \otimes A_3$

Tensor product



$D_5 \otimes A_3$

Tensor product



$D_5 \otimes A_3$

Zamolodchikov periodicity

Theorem (B. Keller, 2013)

Tensor product of **finite** *Dynkin diagrams* $\Longrightarrow$ *the* T*-system is periodic.*

Zamolodchikov periodicity

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- Zamolodchikov (1991): $\Lambda \otimes A_1$ (conjectured);

Zamolodchikov periodicity

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Zamolodchikov periodicity

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- Fomin-Zelevinsky (2003): $\Lambda \otimes A_1$;

Zamolodchikov periodicity

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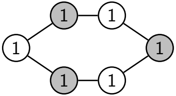
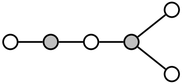
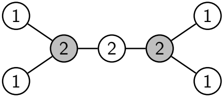
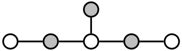
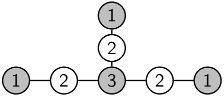
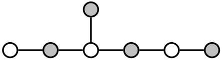
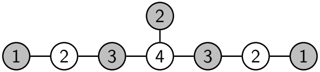
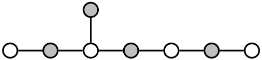
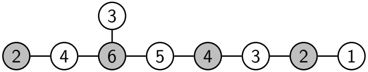
- Ravanini-Tateo-Valleriani (1993): $\Lambda \otimes \Lambda'$ (conjectured);
- Frenkel-Szenes (1995): $A_n \otimes A_1$;
- Fomin-Zelevinsky (2003): $\Lambda \otimes A_1$;
- Volkov (2005): $A_n \otimes A_m$;

ADE Dynkin diagrams

Name	Finite diagram	h	Affine diagram	Name
A_n		$\mathbf{n+1}$		$\hat{A}_{n-1}$
D_n		$\mathbf{2n-2}$		$\hat{D}_{n-1}$
E_6		**12**		$\hat{E}_6$
E_7		**18**		$\hat{E}_7$
E_8		**30**		$\hat{E}_8$

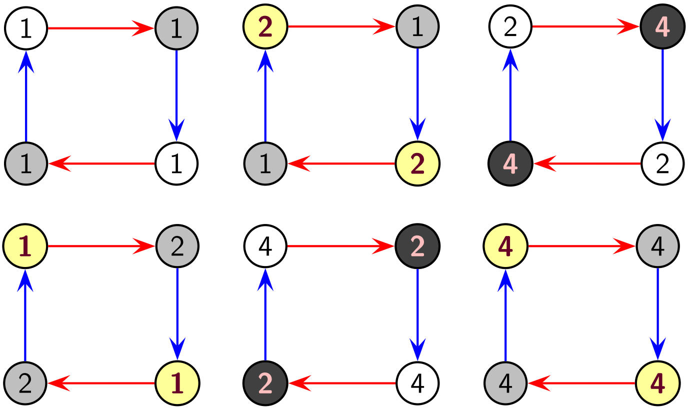
Zamolodchikov periodicity

Theorem (B. Keller, 2013)

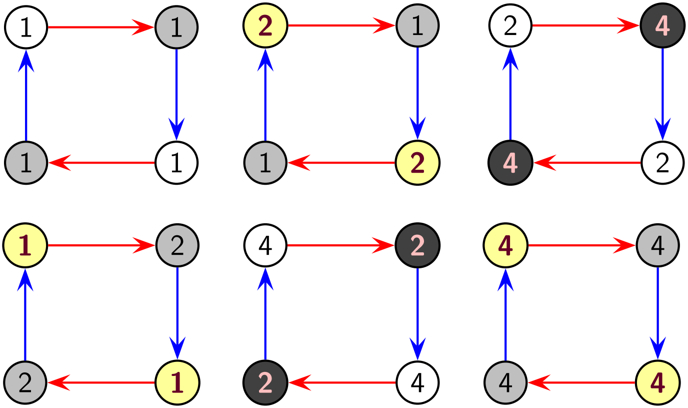
Tensor product of **finite** *Dynkin diagrams* $\Longrightarrow$ *the* T*-system is periodic* **with period dividing**

$$2(h+h').$$

Example: $A_2 \otimes A_2$

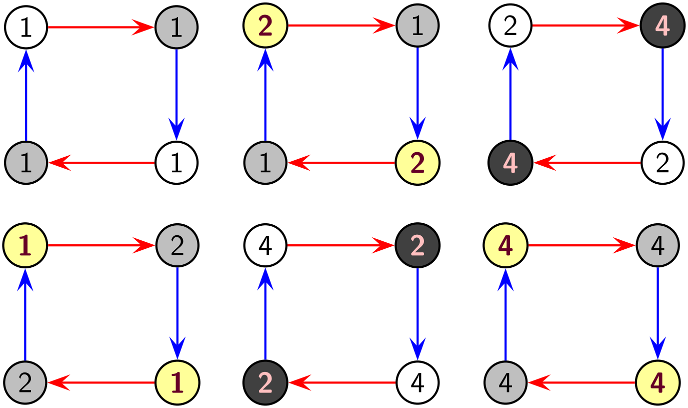


Example: $A_2 \otimes A_2$



6 steps!

Example: $A_2 \otimes A_2$



12
~~6~~ **steps!**

Results

Theorem (G.-Pylyavskyy, 2016)

Periodic $\Longleftrightarrow$ finite $\boxtimes$ finite

Theorem (G.-Pylyavskyy, 2016)

Linearizable $\Longrightarrow$ *affine* $\boxtimes$ *finite or finite* $\boxtimes$ *finite*

Theorem (G.-Pylyavskyy, 2017)

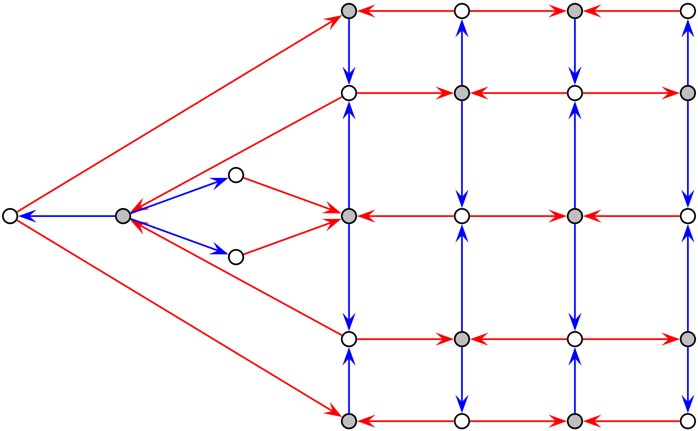
Grows slower than $\exp(\exp(t)) \Longrightarrow$ *affine* $\boxtimes$ *affine, affine* $\boxtimes$ *finite, or finite* $\boxtimes$ *finite*

What is left:

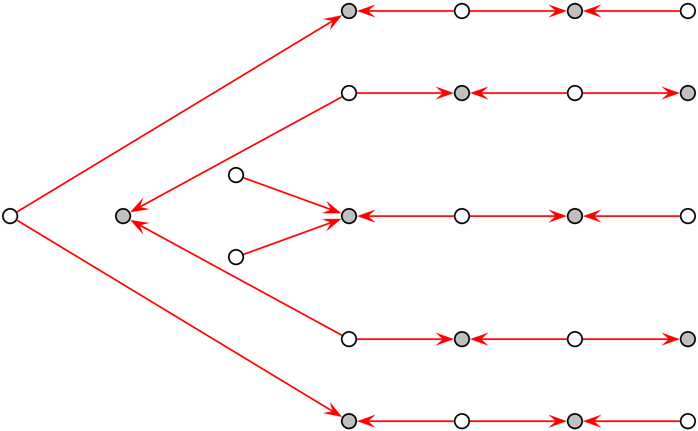
Conjecture (G.-Pylyavskyy, 2017)

- *affine* $\boxtimes$ *finite* $\Longrightarrow$ *linearizable*
- *affine* $\boxtimes$ *affine* $\Longrightarrow$ *grows as* $\exp(t^2)$

Finite ⊠ finite quivers

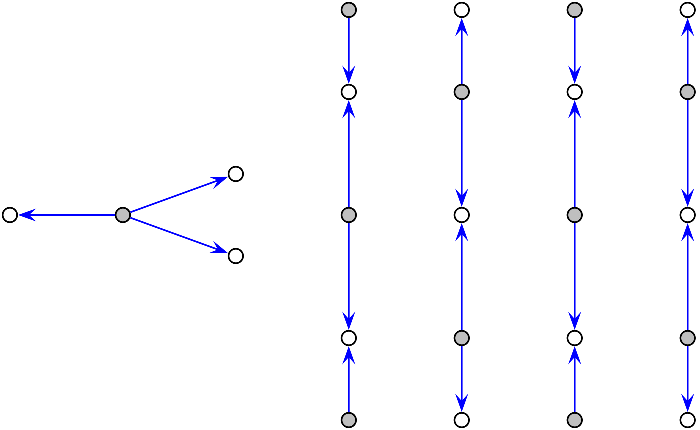


Finite ⊠ finite quivers



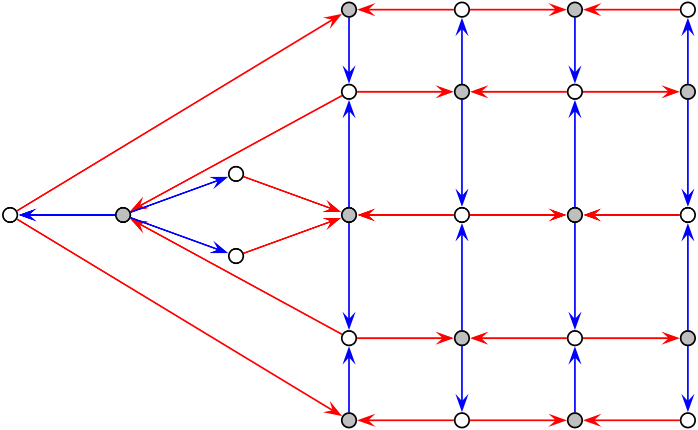
$\mathbf{h} = \mathbf{9} + \mathbf{1} = \mathbf{12} - \mathbf{2} = \mathbf{10}$;

Finite ⊠ finite quivers



$h = 9 + 1 = 12 - 2 = 10;\quad \mathbf{h' = 5 + 1 = 8 - 2 = 6};$

Finite ⊠ finite quivers

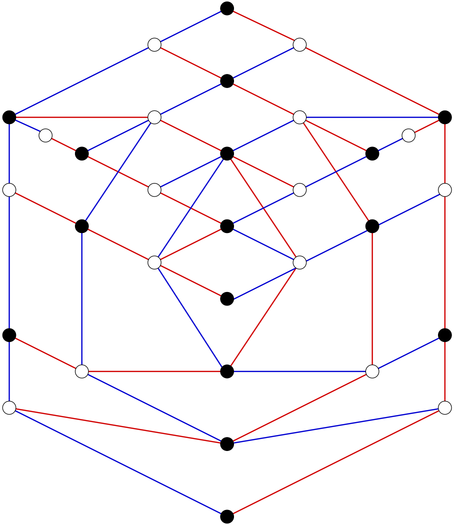


$h = 9 + 1 = 12 - 2 = 10;\quad h' = 5 + 1 = 8 - 2 = 6;\quad$ Period $=$ **32**

Part 3: The classification

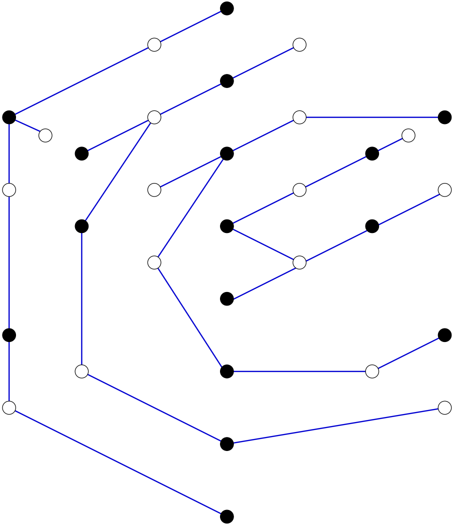
Finite ⊠ finite classification (Stembridge, 2010)

5 infinite families and 11 exceptional quivers



Finite ⊠ finite classification (Stembridge, 2010)

5 infinite families and 11 exceptional quivers



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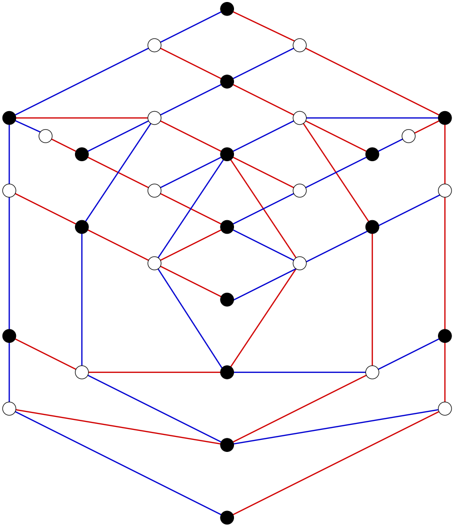
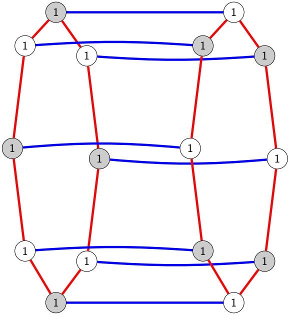
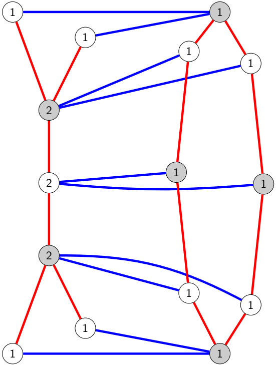
5 infinite families and 11 exceptional quivers



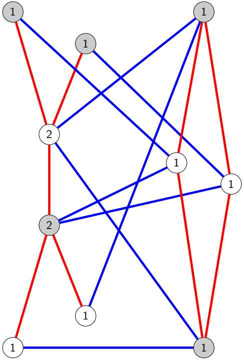
Scaling factor: affine ⊠ finite



$\mathrm{scf} = (1,1)$



$\mathrm{scf} = (1,2)$



$\mathrm{scf} = (1,3)$

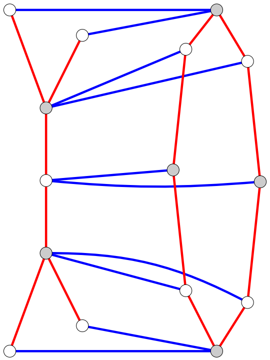
$\hat{A}_7 \text{—} \hat{A}_7$ $\quad$ $\hat{D}_6 \Rrightarrow \hat{A}_7$ $\quad$ $\hat{D}_5 \Rrightarrow \hat{A}_3$

Finite Dynkin diagrams

A_ℓ	$\bullet - \bullet - \cdots - \bullet - \bullet$	B_ℓ	$\bullet - \bullet - \cdots - \bullet \Rightarrow \bullet$
C_ℓ	$\bullet \Rightarrow \bullet - \cdots - \bullet - \bullet$	D_ℓ	$\begin{array}{ccccccccc} & & \bullet & & & & & & \\ & & \mid & & & & & & \\ \bullet & - & \bullet & - & \cdots & - & \bullet & - & \bullet \end{array}$
E_6	$\begin{array}{ccccccccc} & & & & \bullet & & & & \\ & & & & \mid & & & & \\ \bullet & - & \bullet & - & \bullet & - & \bullet & - & \bullet \end{array}$	E_7	$\begin{array}{ccccccccccc} & & & & & & \bullet & & & & \\ & & & & & & \mid & & & & \\ \bullet & - & \bullet & - & \bullet & - & \bullet & - & \bullet & - & \bullet \end{array}$
E_8	$\begin{array}{ccccccccccccc} & & & & \bullet & & & & & & & & \\ & & & & \mid & & & & & & & & \\ \bullet & - & \bullet & - & \bullet & - & \bullet & - & \bullet & - & \bullet & - & \bullet \end{array}$	F_4	$\bullet - \bullet \Rightarrow \bullet - \bullet$
G_2	$\bullet \Rrightarrow \bullet$		

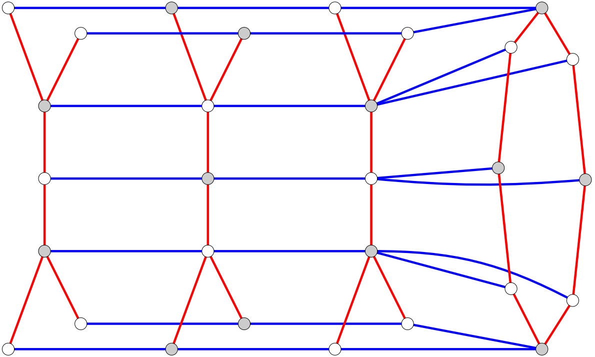
Affine ⊠ finite classification

15 infinite families and 4 exceptional cases



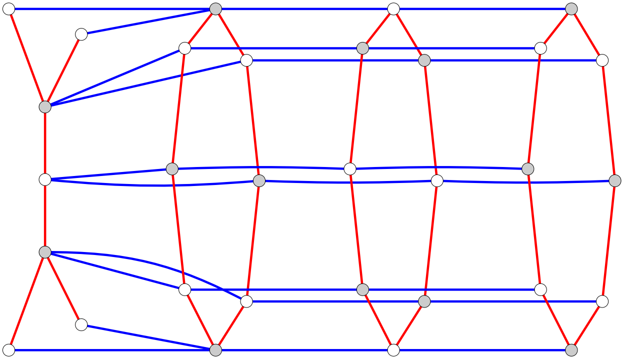
$$\hat{D}_6 \Rrightarrow \hat{A}_7$$

15 infinite families and 4 exceptional cases



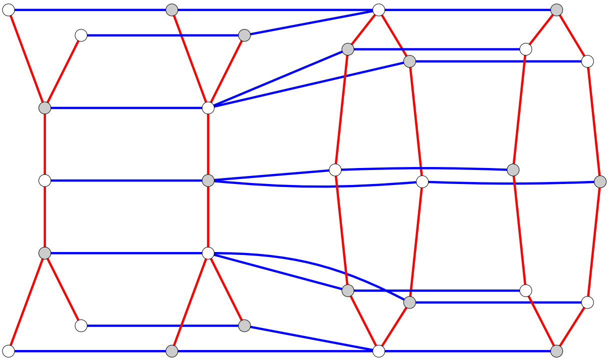
$$\hat{D}_6 — \hat{D}_6 — \hat{D}_6 \Rightarrow \hat{A}_7$$

15 infinite families and 4 exceptional cases



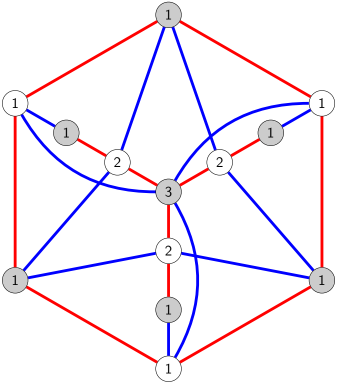
$$\hat{D}_6 \Rightarrow \hat{A}_7 - \hat{A}_7 - \hat{A}_7$$

15 infinite families and 4 exceptional cases

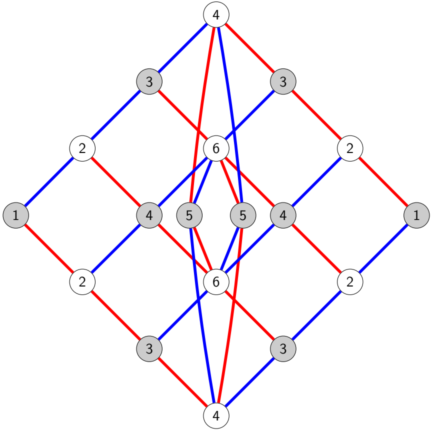


$$\hat{D}_6 - \hat{D}_6 \Rightarrow \hat{A}_7 - \hat{A}_7$$

Scaling factor: affine ⊠ affine



scf $= (1, 4)$

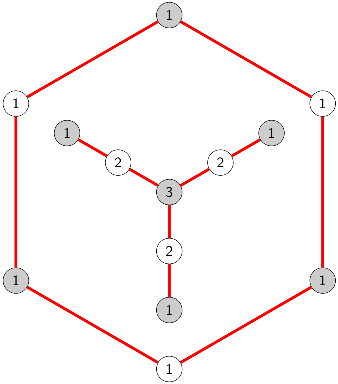


scf $= (2, 2)$

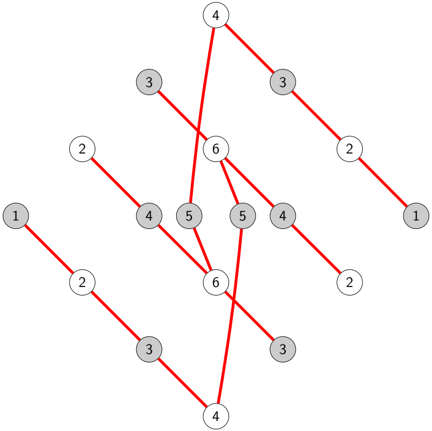
$\hat{E}_6 \Rrightarrow \hat{A}_5$

$\hat{E}_8 \Leftrightarrow \hat{E}_8$

Scaling factor: affine $\boxtimes$ affine



scf $= (1, 4)$

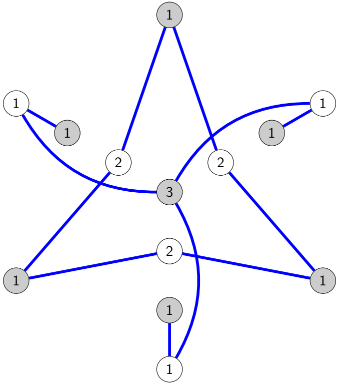


scf $= (2, 2)$

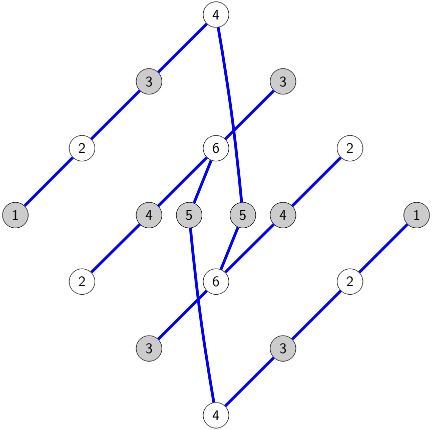
$\hat{E}_6 \Rrightarrow \hat{A}_5$

$\hat{E}_8 \Leftrightarrow \hat{E}_8$

Scaling factor: affine ⊠ affine



scf $= (1, 4)$



scf $= (2, 2)$

$\hat{E}_6 \Rrightarrow \hat{A}_5$

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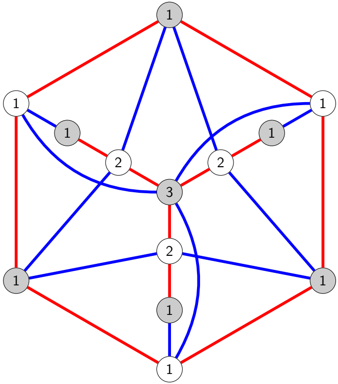
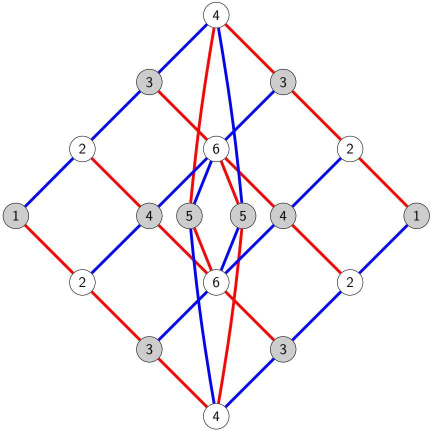


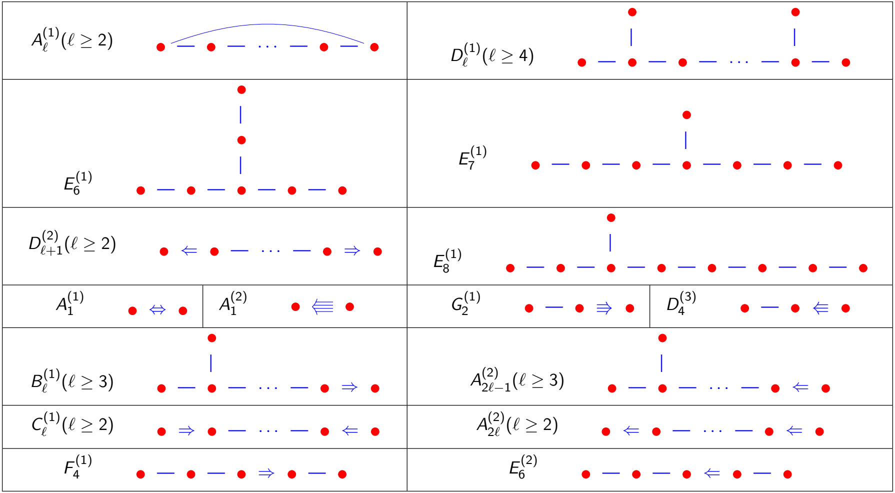
scf $= (1, 4)$



scf $= (2, 2)$

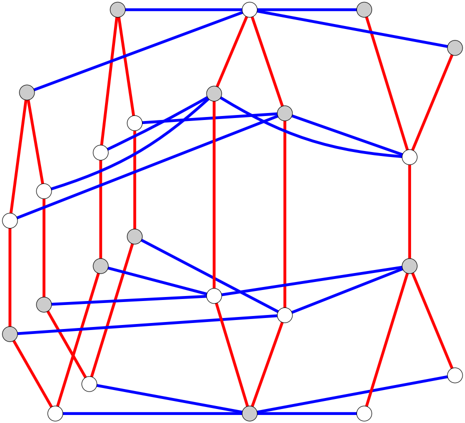
$\hat{E}_6 \Rrightarrow \hat{A}_5$ $\qquad$ $\hat{E}_8 \Leftrightarrow \hat{E}_8$

Affine Kac diagrams



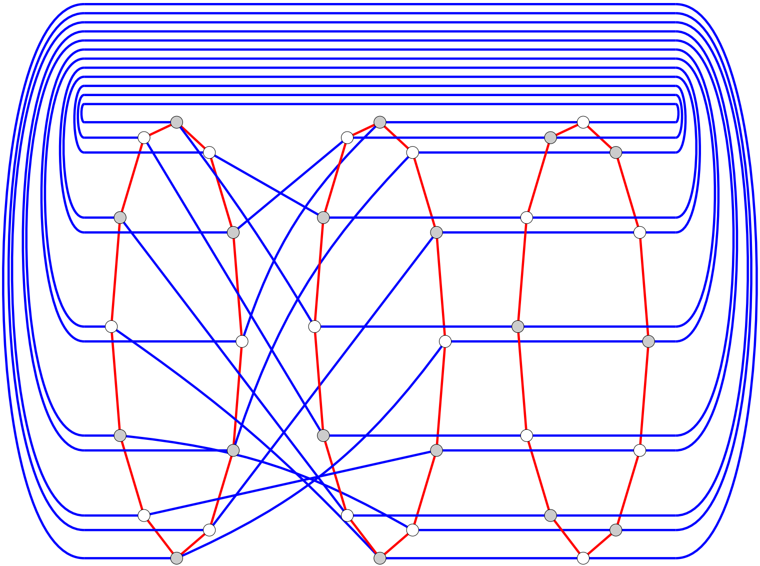
Pairs of affine Kac diagrams

$\hat{D}_4 \Leftarrow \hat{D}_6 - \hat{D}_6 \Rightarrow \hat{D}_4$

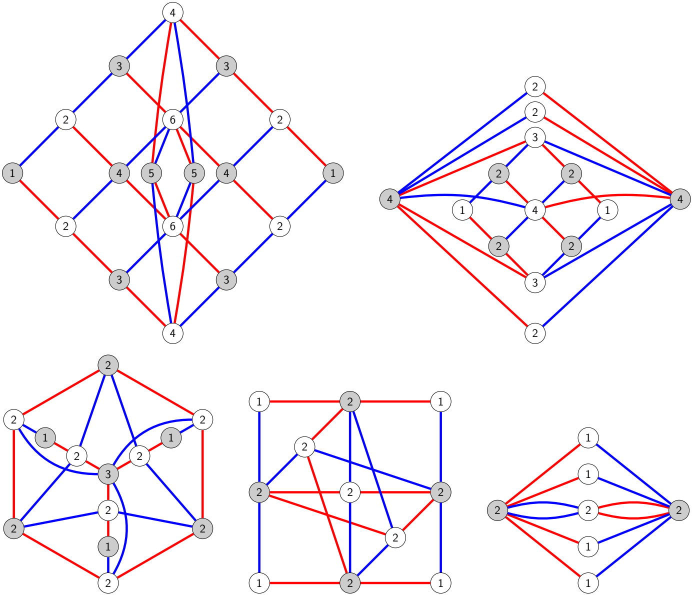


$\hat{A}_{11} \Rightarrow \hat{A}_5 \Leftarrow \hat{D}_5$

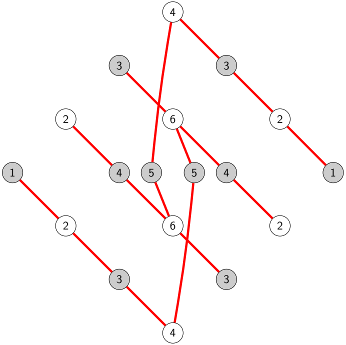
Toric quivers

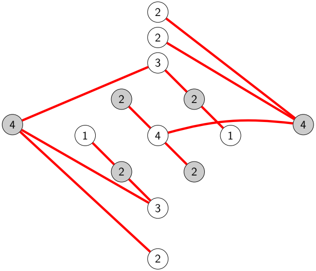


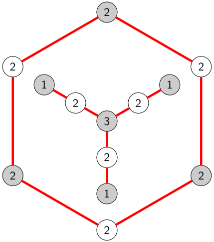
Affine ⊠ affine classification: 40 infinite, 13 exceptional

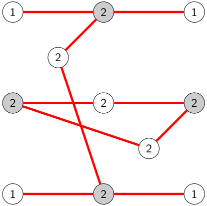


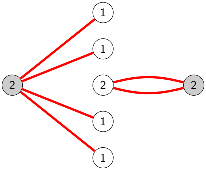
Affine ⊠ affine classification: 40 infinite, 13 exceptional



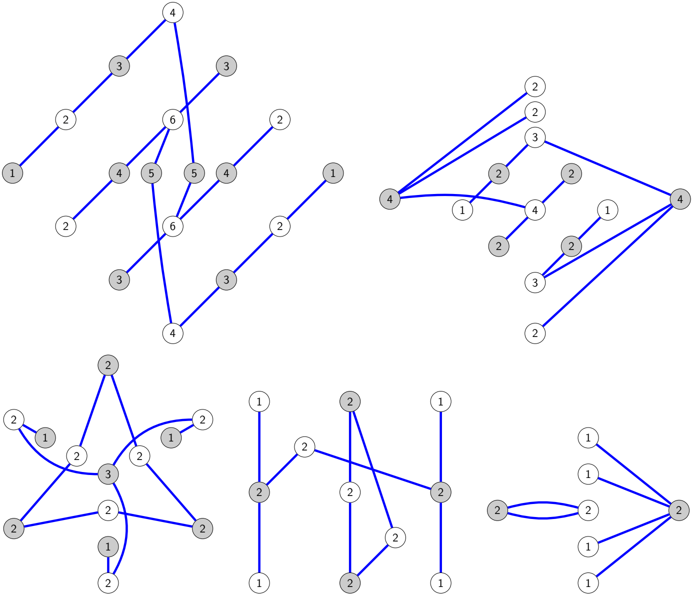




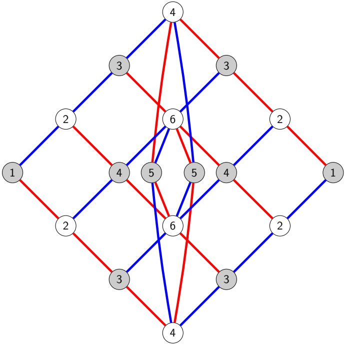


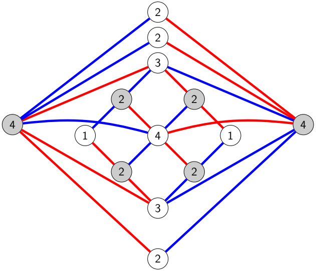


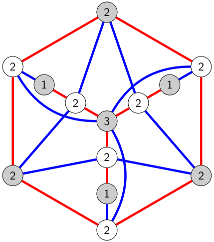
Affine ⊠ affine classification: 40 infinite, 13 exceptional

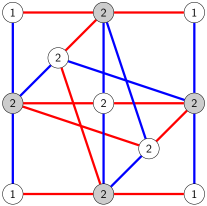


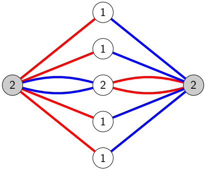
Affine ⊠ affine classification: 40 infinite, 13 exceptional











Results

Theorem (G.-Pylyavskyy, 2016)

Periodic $\iff$ *finite* $\boxtimes$ *finite*

Theorem (G.-Pylyavskyy, 2016)

Linearizable $\Longrightarrow$ *affine* $\boxtimes$ *finite or finite* $\boxtimes$ *finite*

Theorem (G.-Pylyavskyy, 2017)

Grows slower than $\exp(\exp(t)) \Longrightarrow$ **affine** $\boxtimes$ **affine, affine** $\boxtimes$ **finite, or finite** $\boxtimes$ **finite**

What is left:

Conjecture (G.-Pylyavskyy, 2017)

- *affine* $\boxtimes$ *finite* $\Longrightarrow$ *linearizable*
- *affine* $\boxtimes$ *affine* $\Longrightarrow$ *grows as* $\exp(t^2)$

Results

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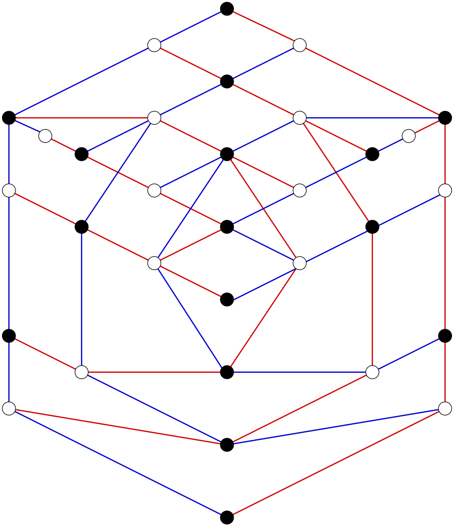
Ocneanu graphs (2001)



Ocneanu graphs (2001)

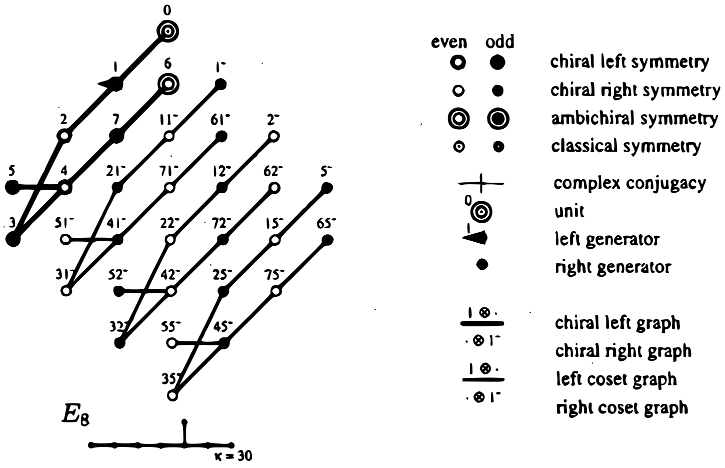


Fig. 23. Quantum symmetries for Coxeter graphs E_8.

Ocneanu graphs (2001)

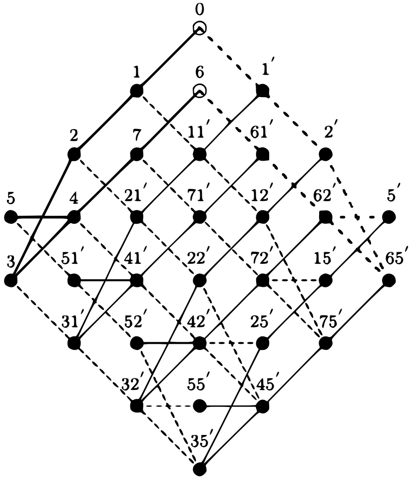


Figure 8: The E_8 Ocneanu graph

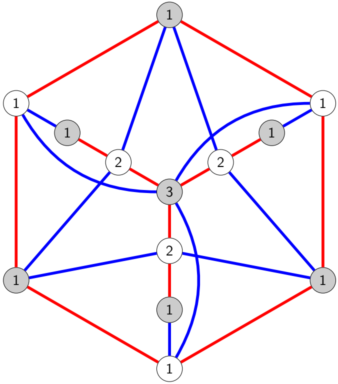
McKay correspondence

Theorem (J. McKay, 1980)

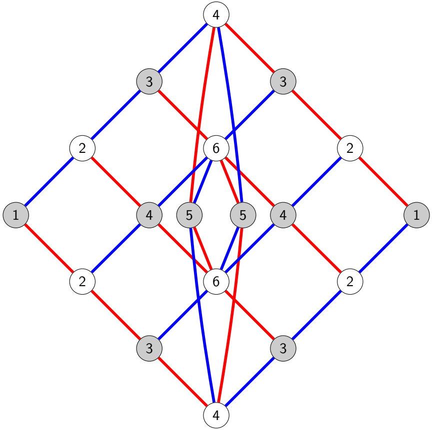
There is a one-to-one correspondence between finite subgroups of $SU(2)$ and affine ADE Dynkin diagrams:

Dynkin diagram	Binary polyhedral group
$\hat{A}_n$	Binary cyclic group
$\hat{D}_n$	Binary dihedral group
$\hat{E}_6$	Binary tetrahedral group
$\hat{E}_7$	Binary octahedral group
$\hat{E}_8$	Binary icosahedral group

McKay correspondence



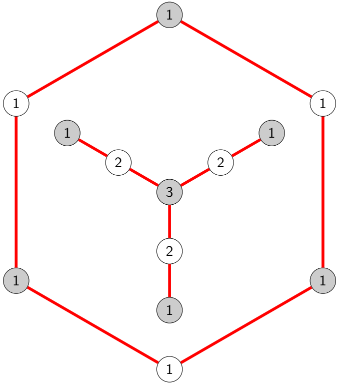
scf $= (1,4)$



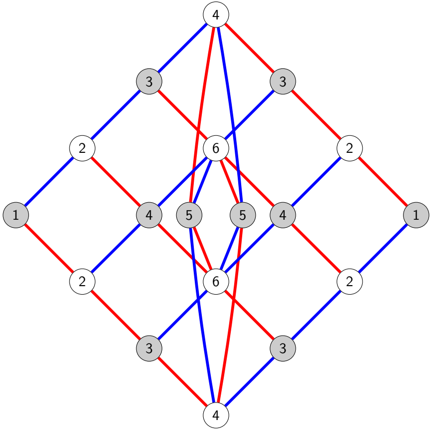
scf $= (2,2)$

$$\hat{E}_6 \Rrightarrow \hat{A}_5 \qquad \hat{E}_8 \Leftrightarrow \hat{E}_8$$

McKay correspondence



$\mathrm{scf} = (1, 4)$



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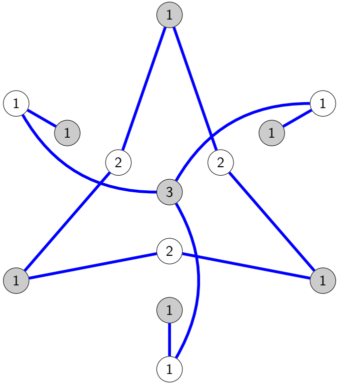

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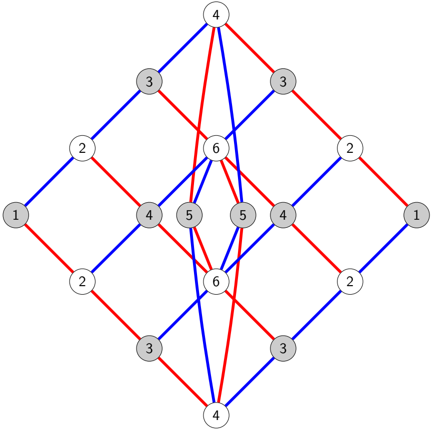

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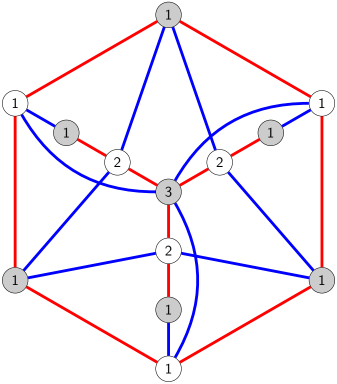
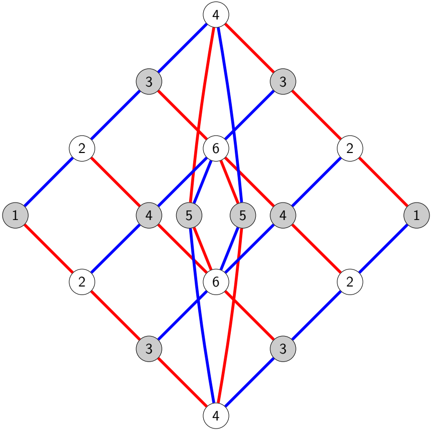
McKay correspondence



$\mathrm{scf} = (1, 4)$



$\mathrm{scf} = (2, 2)$

$$\hat{E}_6 \Rrightarrow \hat{A}_5 \qquad \hat{E}_8 \Leftrightarrow \hat{E}_8$$

Bibliography

Slides: `http://math.mit.edu/~galashin/slides/umich.pdf`

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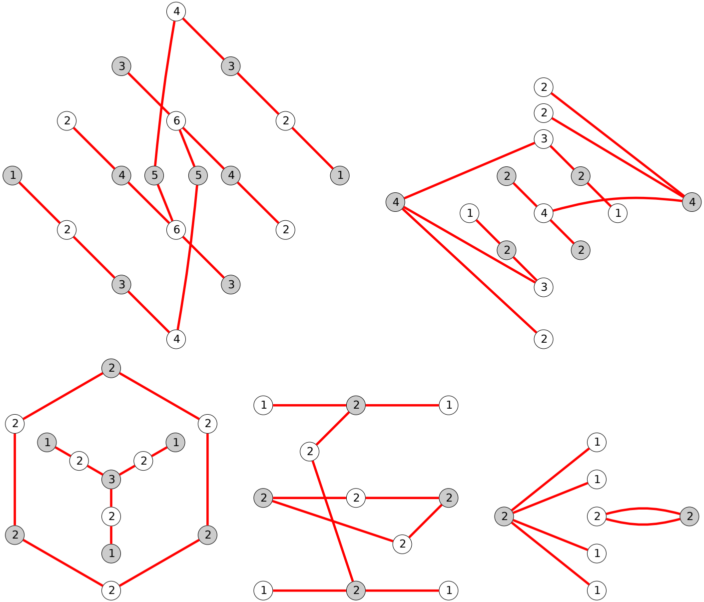
Thank you!



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